3 qubits

Find QFT for $|x\rangle = |x_2 x_1 x_0\rangle$

$$|\tilde{x}\rangle = \frac{1}{\sqrt{8}} \sum_{y_2, y_1, y_0 \in \{0,1\}} e^{2\pi i x(4y_2 + 2y_1 + y_0)/8} |y_2 y_1 y_0\rangle$$

$$= \frac{1}{\sqrt{2}} \sum_{y_2 \in 0,1} e^{2\pi i x(4y_2/8)} |y_2\rangle \otimes \frac{1}{\sqrt{2}} \sum_{y_1 \in 0,1} e^{2\pi i x(2y_1/8)} |y_1\rangle$$

$$\otimes \frac{1}{\sqrt{2}} \sum_{y_0 \in 0,1} e^{2\pi i x(y_0/8)}$$

$$\frac{1}{\sqrt{2}}\sum_{y_2 \in 0,1} e^{2\pi i x(4y_2/8)} |y_2\rangle$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x/2} |1\rangle \right].$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{\pi i x} |1\rangle \right]$$

$$x = 4x_2 + 2x_1 + x_0$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x_0/2} |1\rangle \right]$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + (-1)^{x_0} |1\rangle \right].$$

2nd term

$$\frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x/4}|1\rangle\right]$$

$$x = 4x_2 + 2x_1 + x_0.$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + (-1)^{x_1}\, \underline{e^{2\pi i x_0/4}}\, |1\rangle.\right]$$

Phase Rotation

$2\pi x_0/4$.

$B_{01}^{x_0}$

Hadamard.

$$B_{jk} = \frac{2\pi}{2^{k-j+1}}$$

Third term

$$\sum_{y_0=0}^{1} e^{2\pi i(4x_2+2x_1+x_0)\cdot y_0/8} |y_0\rangle$$

$$= \left[|0\rangle + e^{2\pi i(4x_2+2x_1+x_0)/8} |1\rangle \right]$$

$$= \left[|0\rangle + (-1)^{x_2} \underbrace{e^{2\pi i x_1/4}}_{B_{12}^{x_1}} \cdot \underbrace{e^{2\pi i x_0/8}}_{B_{02}^{x_0}} |1\rangle \right]$$

$$B_{jk} = \frac{2\pi}{2^{k-j+1}}$$

$$\frac{1}{\sqrt{2}}\left[|0\rangle + (-1)^{x_0}|1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[B_{01}^{x_0}\left(|0\rangle + (-1)^{x_1}|1\rangle\right)\right]$$

$$\otimes \frac{1}{\sqrt{2}}\left[B_{01}^{x_0}\, B_{12}^{x_1}\left(|0\rangle + (-1)^{x_2}|1\rangle\right)\right]$$