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$$|\psi\rangle = \sum_x a_x |x\rangle.$$

$$|\psi'\rangle = U|\psi\rangle = \sum_x a_x U|x\rangle$$

$$= \sum_y \alpha_y |\psi\rangle.$$

$$\alpha_y = \frac{1}{\sqrt{N}} \sum_x \omega^{xy} a_x \qquad \omega = e^{2\pi i/N}.$$

$$U = \sum_{yz} \frac{e^{2\pi i yz/N}}{\sqrt{N}} |y\rangle\langle z|$$

$$|\tilde{x}\rangle = U_f |x\rangle$$

$$= \frac{1}{\sqrt{N}} \sum_{y=0}^{N-1} e^{2\pi i xy/N} |y\rangle$$

$n = 1 \qquad N = 2$

$$|\tilde{x}\rangle = \frac{1}{\sqrt{2}} \sum_{y \in 0,1} e^{2\pi i xy/2} |y\rangle$$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle + e^{2\pi i x/2} |1\rangle \right]$$

$$x/2 = 0.x$$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle + e^{2\pi i (0.x)} |1\rangle \right]$$

Hadamard.

$n=2 \qquad N=4 \quad (2\ \text{qubit})$

$$|\tilde{x}\rangle = \frac{1}{2}\sum_{y=0}^{3} e^{2\pi i xy/4}\,|y\rangle.$$

$$= \frac{1}{2}\sum_{y_1,y_0} e^{2\pi i x(2y_1+y_0)/4}\,|y_1,y_0\rangle \qquad |y\rangle = |y_1 y_0\rangle$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x/2}|1\rangle\right] \times \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x/2^2}|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i(0.x_0)}\right] * \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i(0.x_1x_0)}|1\rangle\right]$$

$x = 2x_1 + x_0.$

$$|\tilde{x}\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i(0.x_0)}|1\rangle\right]$$

$$\times \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i(0.x_1x_0)}|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[\underbrace{|0\rangle + (-1)^{x_0}|1\rangle}_{\text{Hadamard Gate}}\right] \times \frac{1}{\sqrt{2}}\left[|0\rangle + (-1)^{x_1} e^{2\pi i x_0/4}|1\rangle\right]$$

$\downarrow$

Hadamard followed by a rotation

$$\frac{2\pi x_0}{4}$$

Controlled rotation

Controled B_{jk}.

$$B_{jk} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^{k-j+1}} \end{pmatrix}$$

$$B_{jk}\,|x,y\rangle = \left(e^{\frac{2\pi i}{2^{k-j+1}}xy}\right)|x,y\rangle$$

$$= |x,y\rangle \quad \text{if} \quad x = 0$$

$$= \exp\left(\frac{2\pi i}{2^{k-j+1}}\right)|y\rangle$$

$$\tilde{x} = \frac{1}{2}\Big[\underbrace{|0\rangle + (-1)^{x_0}|1\rangle}_{\Downarrow\ (-1)^{x_0}}\Big]\, B_{12}^{0}\, \Big[\underbrace{|0\rangle + (-1)^{x_1}|1\rangle}_{(-1)^{x_1}.}\Big]$$

$$= \frac{1}{2}\left[U_H\,|x_0\rangle\right] \otimes B_{12}^{0}\, U_H\,|x_1\rangle.$$

$$\tilde{x} = \frac{1}{2}\left[U_H\,|x_0\rangle\right] \otimes B_{12}^{0}\, U_H\,|x_1\rangle.$$

$$= \frac{1}{2}\left[(U_H \otimes I)\, B_{12}^{0}\, (I \otimes U_H)\right]|x_0\, x_1\rangle$$

$$= \frac{1}{2}\left[(U_H \otimes I)\, B_{12}^{0}\, (I \otimes U_H)\right] \mathrm{Swap}\,|x_1\, x_0\rangle$$

$$= \frac{1}{2}\Big[\,(U_H \otimes I)\, B_{12}\, (I \otimes U_H)\,\Big]$$

$n=2 \quad N=4.$

$$|\tilde{x}\rangle = \frac{1}{2}\sum_{y=0}^{3} e^{2\pi i x y/4} |y\rangle$$

$y = 2y_1 + y_0$

$y_1, y_0 \in 0, 1$

$|y\rangle = |y_1 y_0\rangle$

$$= \frac{1}{2}\sum_{y_1, y_0} e^{2\pi i x(2y_1+y_0)/4} |y_1 y_0\rangle$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 \sum_{y_1 \in 0,1} e^{2\pi i x(2y_1/4)} |y_1\rangle \otimes \sum_{y_0=0,1} e^{2\pi i x. y_0/4}.$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x/2}|1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x/4}|1\rangle\right]$$

$$x = 2x_1 + x_0$$

$$e^{2\pi i(2x_1+x_0)/2}$$

$$= \underline{e^{2\pi i x_1}} \cdot e^{2\pi i x_0/2}.$$

$$\frac{1}{\sqrt{2}}\left[\,|0\rangle + e^{2\pi i x_0/2}|1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i(2x_1+x_0)/4}|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i x_0/2}|1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + (-1)^{x_1} e^{2\pi i x_0/4}|1\rangle\right]$$

$$x_0/2 = 0.x_0$$

$$e^{2\pi i \underline{x_1}/2} e^{2\pi i x_0/4} = e^{0.x_1x_0}$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i(0.x_0)}|1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + e^{2\pi i(0.x_1x_0)}|1\rangle\right.$$

$$\frac{k}{r} \leq \frac{f(2)}{f(M)} \leq \frac{k+1}{r}$$

$$M^k \leq 2^r \leq M^{k+1}$$

$$\log(M^k) \leq \log 2^r \leq \log(M^{k+1})$$

$$k \log M \leq r \log 2 \leq (k+1) \log M.$$

$$\frac{k}{r} \leq \frac{\log 2}{\log M} \leq \frac{k+1}{r}$$