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Lec. 24. 3/6/2016

$f(x)$ of discrete variable

$x \in S_n \{0, 1, 2, \cdots N-1\}$

DIT

$$\tilde{f}(y) = \sum_{x=0}^{N-1} K(\underline{y, x})\, f(x)$$

If K is unitary

$$f(x) = \sum_{y=0}^{N-1} K^{+}(x, y)\, \tilde{f}(y)$$

(A specific Kernel)

$$U|x\rangle = \sum_y U(x,y)\,|y\rangle$$

$$\equiv \sum_y K(x,y)\, f(y)$$

$$K(x,y) = \frac{1}{\sqrt{N}}\, e^{2\pi i xy/N}.$$

$$= \frac{1}{\sqrt{N}}\, \omega_n^{xy} \rightarrow \text{Fourier Transform}$$

<u>QFT</u>

$$f(x+P) = f(x)$$

↳ P is the period

↳ Periodicity.

First Register

$$\frac{1}{\sqrt{8}}\Big[\,|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle\,\Big].$$

$$= \frac{1}{\sqrt{8}}\Big[\,|0\rangle + |1\rangle + |2\rangle + |3\rangle + |4\rangle + |5\rangle + |6\rangle + |7\rangle\,\Big]$$

Second Register $|000\rangle \equiv |0\rangle$

Oracle Computes f(x)

$$|\psi\rangle = U_f \frac{1}{\sqrt{8}} \sum_{x=0}^{7} |x\rangle|0\rangle$$

Oracle Computing f(x)

$$= \frac{1}{\sqrt{8}} \sum_{x=0}^{7} |x\rangle|f(x)\rangle$$

Entangled.

$$|\psi\rangle = \frac{1}{\sqrt{8}} \Big[|0,f(0)\rangle + |1,f(1)\rangle + |2,f(2)\rangle + |3,f(3)\rangle + |4,f(4)\rangle + |5,f(5)\rangle + |6,f(6)\rangle + |7,f(7)\rangle \Big]$$

QFT is applied on 1st Reg.

QFT $|\psi\rangle$

$$|\psi'\rangle = \frac{1}{8}\sum_{x=0}^{7}\sum_{y=0}^{7} e^{2\pi i xy/8} |y, f(x)\rangle$$

$$|\psi'\rangle = \frac{1}{8}|0\rangle\left[|f(0)\rangle + f(1) + \dots + |f(7)\rangle\right].$$

$$+ \frac{1}{8}|1\rangle\left[|f(0)\rangle + e^{2\pi i/8}|f(1)\rangle + e^{4\pi i/8}|f(2)\rangle + \dots + e^{7\times 2\pi i/8}|f(7)\rangle\right]$$

$$+ \dots$$

$$+ \frac{1}{8}|7\rangle\left[|f(0)\rangle + e^{14\pi i/8}|f(1)\rangle + \dots + e^{14\pi i\times 7/8}|f(7)\rangle\right].$$

$$f(x+P) = f(x) \qquad P = 2$$

$$f(0) = f(2) = f(4) = f(6) = a$$
$$f(1) = f(3) = f(5) = f(7) = b.$$

First term $|y\rangle = |a\rangle$.

$$\frac{1}{8}|0\rangle \times \left[|f(0)\rangle + |f(1)\rangle + \cdots + |f(7)\rangle\right]$$

$$= \frac{1}{8}|0\rangle \times \left[4|a\rangle + 4|b\rangle\right]$$

$P = 2$ 7**

$$f(0) = f(2) = f(4) = f(6) = a$$
$$f(1) = f(3) = f(5) = f(7) = b$$

$$|\psi'\rangle = \frac{1}{8} \times 4 \left(|a\rangle + |b\rangle \right)$$

From $|y\rangle = |0\rangle$.

<u>$|y\rangle = 1$ term</u>

$$\frac{1}{8} \left[|a\rangle + e^{2\pi i/8} |b\rangle + e^{4\pi i/8} |a\rangle + e^{6\pi i/8} |b\rangle + \dots \right]$$

$$= \frac{1}{8} \Big[\left(1 + e^{4\pi i/8} + e^{8\pi i/8} + e^{12\pi i/8} \right) |a\rangle + \left(e^{2\pi i/8} + e^{6\pi i/8} + e^{10\pi i/8} + e^{14\pi i/8} \right) |b\rangle.$$

$= 0$

$$\frac{1}{8}|1\rangle \Big[|a\rangle \left(1 + e^{2\times 2\pi i/8} + e^{4\times 2\pi i/8} + e^{6\times 2\pi i/8}\right)^{8}$$

$$+ |b\rangle \left(e^{1\times 2\pi i/8} + e^{3\times 2\pi i/8} + e^{5\times 2\pi i/8} + e^{7\times 2\pi i/8} \right)\Big] ?$$

$$|a\rangle : \quad 1 + e^{\pi i/2} + e^{\pi i} + e^{12\pi i/8}$$

$$= 1 + i + (-1) + (-i) = 0$$

$$|b\rangle : \quad \frac{1+i}{\sqrt{2}} + \frac{-1+i}{\sqrt{2}} + \frac{-1-i}{\sqrt{2}} + \frac{1-i}{\sqrt{2}}$$

$$= 0$$

For $y = |4\rangle$

$$e^{4.0.2\pi i/8} + e^{4.2.2\pi i/8} + e^{4.4.2\pi i/8} + e^{4.6.2\pi i/8}$$

$$= 1 + e^{2\pi i \cancel{\times 2}} + e^{4\pi i} + e^{6\pi i \times \cancel{7}}$$

$$= 4.$$

$$|\psi'\rangle = |0\rangle\left[\frac{1}{2}|a\rangle + \frac{1}{2}|b\rangle\right] + |4\rangle\left[\frac{1}{2}|a\rangle - \frac{1}{2}|b\rangle\right].$$

$$|4\rangle \cdot \frac{1}{8}\Big[|a\rangle\big(e^{4.0.2\pi i/8} + e^{4.2.2\pi i/8} + e^{4.4.2\pi i/8} + e^{4.6.2\pi i/8}\big)$$

$$+ |b\rangle$$

$$= \frac{1}{8}\Big[|a\rangle(4) + |b\rangle(-4)\Big]$$

$$e^{4.1.2\pi i/8}$$