

$$|u\rangle = \frac{1}{\sqrt{N-1}} \sum_{x \neq \omega} |x\rangle$$

$$|s\rangle = \frac{1}{\sqrt{N}} |\omega\rangle + \sqrt{\frac{N-1}{N}} |u\rangle$$

jth iteration

$|u_j\rangle$ $|s_j\rangle$

$$\begin{pmatrix} u_{j+1} \\ u_{j+1} \\ \vdots \\ u_{j+1} \\ \vdots \\ u_{j+1} \end{pmatrix} = - \begin{pmatrix} u_j \\ u_j \\ \vdots \\ u_j \\ \vdots \\ u_j \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{pmatrix} \begin{pmatrix} u_j \\ u_j \\ \vdots \\ u_j \\ \vdots \\ u_j \end{pmatrix}$$

$$\omega_{j+1} = \left(\frac{2}{N}-1\right)\omega_j + \frac{2}{N}(N-1)u_j$$

$$u_{j+1} = \frac{2}{N}\omega_j + \frac{N-2}{N}u_j$$

$$\omega_{j+1} = \left(1 - \frac{2}{N}\right)\omega_j + \frac{2}{N}(N-1)u_j$$

$$u_{j+1} = \frac{N-2}{N}\omega_j + \frac{2}{N}u_j$$

$$C_j = \sqrt{N-1} \mu_j$$

$$\begin{pmatrix} \omega_{j+1} \\ \omega_j \end{pmatrix} = \begin{pmatrix} 1 - \frac{2}{N} & \frac{2}{N} \sqrt{N-1} \\ \frac{2}{N} \sqrt{N-1} & 1 - \frac{2}{N} \end{pmatrix} \begin{pmatrix} \omega_j \\ C_j \end{pmatrix}$$

$$\frac{1}{2} = \sin \theta$$

$$\cos \theta = \sqrt{1 - \frac{1}{N}}$$

$$\begin{pmatrix} \omega_{j+1} \\ \omega_j \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \omega_j \\ C_j \end{pmatrix}$$

$$\begin{aligned} &= 2 \times \frac{1}{2} \times \frac{\sqrt{N-1}}{2} \\ &= \frac{2}{2} \sqrt{N-1} \\ &= \sqrt{N-1} \end{aligned}$$

$$\begin{pmatrix} \omega_{j+1} \\ \mu_{j+1} \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \omega_j \\ \mu_j \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \omega_1 \\ \mu_1 \end{pmatrix}$$

$$= \begin{pmatrix} \sin(2j+1)\theta \\ \cos(2j+1)\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}$$

$$D = -I + \frac{2J}{N} \quad N=8$$

$$D = \begin{pmatrix} -0.75 & 0.25 & 0.25 & \dots & 0.25 \\ 0.25 & -0.75 & & & \\ \vdots & & \ddots & & \\ 0.25 & 0.25 & & & -0.75 \end{pmatrix}$$

$$DT = \begin{pmatrix} -.75 & .25 & .25 & -.25 \\ \vdots & & & \\ .25 & .25 & .25 & +.75 \end{pmatrix}$$

DT

$$\begin{pmatrix} u_1 \\ u_1 \\ u_1 \\ w \\ u_1 \\ u_1 \\ \vdots \end{pmatrix} =$$

$$\begin{pmatrix} 0.75u - 0.25w \\ 0.75u - 0.25w \\ 0.75u - 0.25w \\ 1.75u + 0.75w \\ 0.75u - 0.25w \\ \vdots \end{pmatrix}$$