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L-21
Date-1-6-

$$R_G = U_s \underline{U_w}.$$

Maximum Number of iterations

$$\underline{m \approx \frac{\pi}{4}\sqrt{N}.}$$

$\angle\, |s\rangle, |w\rangle$

$$\frac{\pi}{2} - (2m+1)\theta .$$

Amplitude of $|w\rangle$ in $|s\rangle$

$$\sin (2m+1)\theta = \sin\left[\left(\frac{\pi}{2}\sqrt{N} + 1\right)\cdot \frac{1}{\sqrt{N}}\right].$$

$$= \sin\left(\frac{\pi}{2} + \frac{1}{\sqrt{N}}\right)$$

$$= \cos\frac{1}{\sqrt{N}} \approx 1 - \frac{1}{2N} .$$

# Diffusion Operator.

$$D_{ij} = \begin{cases} -1 + \frac{2}{N} & \text{if } i = j \\ \frac{2}{N} & \text{if } i \neq j \end{cases}$$

$$D = -I + \frac{2J}{N}$$ ; $J$ is a matrix whose each element is 1

$$\left(\frac{J}{N}\right)^2 = \frac{J}{N}.$$

$$|\psi\rangle = \sum_x a_x |x\rangle$$

$$\frac{J}{N}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} \bar{a} \\ \bar{a} \\ \vdots \\ \bar{a} \end{pmatrix}$$

$$D = \left(-I + \frac{2J}{N}\right)\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} = \sum (2\bar{a} - a_x)|x\rangle.$$

$$D = WRW.$$

W : Walsh-Hadamard Transform

R : Selective phase rotation.

$$H^{\otimes n} |x\rangle = \frac{1}{\sqrt{N}} \sum_{x} (-1)^{x \cdot y} |y\rangle$$

$$W_{ij} = \frac{1}{\sqrt{N}} (-1)^{i \cdot j}$$

$$R_{ij} = (2\delta_{i0} - 1)\delta_{ij}$$

$$D = -I + \frac{2J}{N}$$

$$\left(-I + \frac{2J}{N}\right)\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} -a_1 + 2\bar{a} \\ -a_2 + 2\bar{a} \\ \vdots \\ -a_N + 2\bar{a} \end{pmatrix}$$

$$D|\psi\rangle = \sum_x (2\bar{a} - a_x)|x\rangle$$