Prof. D. K. Ghosh
L-20
Date-1-6-16

$$R_G = U_s U_w$$

$$|\psi\rangle = \sum_x a_x |x\rangle \quad \text{arbitrary.}$$

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_x |x\rangle \quad \text{Standard}$$

$$\langle s|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x x'} a_x \cancel{|x\rangle\langle x'|} \underbrace{\langle x|x'\rangle}_{\delta_{xx'}} = \frac{1}{\sqrt{N}} \sum_x a_x$$

$$\bar{a} = \frac{1}{N} \sum_x a_x \quad \text{Mean Amplitude}$$

$$\langle s|\psi\rangle = \sqrt{N}\,\bar{a}$$

$$U_s|\psi\rangle = \left(2|s\rangle\langle s| - I\right)\overbrace{\sum_x a_x|x\rangle}$$

$$= 2|s\rangle\langle s|\psi\rangle - |\psi\rangle$$

$$= 2\sqrt{N}\,\bar{a}\,|s\rangle - |\psi\rangle$$

$$= \sum_x \left(2\bar{a} - a_x\right)|x\rangle$$

$$|\psi\rangle = \sum_x a_x |x\rangle$$

$$U_s |\psi\rangle = \sum_x (2\bar{a} - a_x) |x\rangle$$

Amplitude w.r. to mean

$a_x - \bar{a}$

gets inverted to $\bar{a} - a_x$.

$N = 8$ $\qquad\qquad$ $\omega = 4$

In $|s\rangle$ $\qquad$ $\frac{1}{\sqrt{8}}$

U_ω inverts the amplitude of ω alone : $\omega : -\frac{1}{\sqrt{8}}$

$$\text{Mean} = \frac{1}{8}\left[\frac{7}{\sqrt{8}} - \frac{1}{\sqrt{8}}\right] = \frac{3}{8\sqrt{2}}$$

Application of U_s. : New amplitude $2\bar{a} - a_x$.

Mean $= \frac{3}{8\sqrt{2}}$

unmarked states $u : \frac{1}{\sqrt{8}}$

$w : -\frac{1}{\sqrt{8}}$

Every unmarked state

$$\left(\frac{3}{4\sqrt{2}} - \frac{1}{2\sqrt{2}} \right) = \frac{1}{4\sqrt{2}}$$

Marked State

$$\left(\frac{3}{4\sqrt{2}} + \frac{1}{2\sqrt{2}} \right) = \frac{5}{4\sqrt{2}}$$

$$m \cdot 2\theta \cong \frac{\pi}{2} - \theta$$

$$m = \frac{\pi}{4\theta} - \frac{1}{2}$$

$$\theta \approx \sin\theta = \frac{1}{\sqrt{N}}$$

$$m = \frac{\pi\sqrt{N}}{4} \quad \text{Quadratic}$$