Needle in Haystack. integers k.

$$0 \leq k \leq 2^n - 1$$

$$\boxed{2^n = N}$$

such that $f(k) = 0$ for $\forall k \neq \omega$

$$f(\omega) = 1$$

If $y = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

Target $y \oplus f_w(x)$

$(-1)^{f_w(x)} |x\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

If $f_w(x) = 1$ — 2nd Register.

Sign of 1st Register flipped

$$U_w = I - 2|w\rangle\langle w|$$

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

$|s\rangle$: Standard state

$|x\rangle$: Computational basis

$$\langle w|s\rangle = \frac{1}{\sqrt{N}}.$$

$$U_\omega = I - 2|\omega\rangle\langle\omega|$$

Reflection Operator

$$U_s = 2|s\rangle\langle s| - I$$

$$|\psi\rangle = |\psi_n\rangle + |\psi_\perp\rangle$$

Component parallel to $|s\rangle$ is unchanged

Component $\perp^{ar}$ to $|s\rangle$ will change sign.

Grover Operator. $\boxed{R_G = U_s U_w}$

$$U_w = I - 2|w\rangle\langle w|$$

$$U_s = 2|s\rangle\langle s| - I$$

$$R_G = U_s U_w .$$