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Deutsch Algorithm

Input : One qubit. 0/1

Output : f(0) or f(1).

Types of functions.

(i) Constant Function.

$$f(0) = f(1) = \begin{cases} 0 \\ 1 \end{cases}$$

(ii) Balanced Function

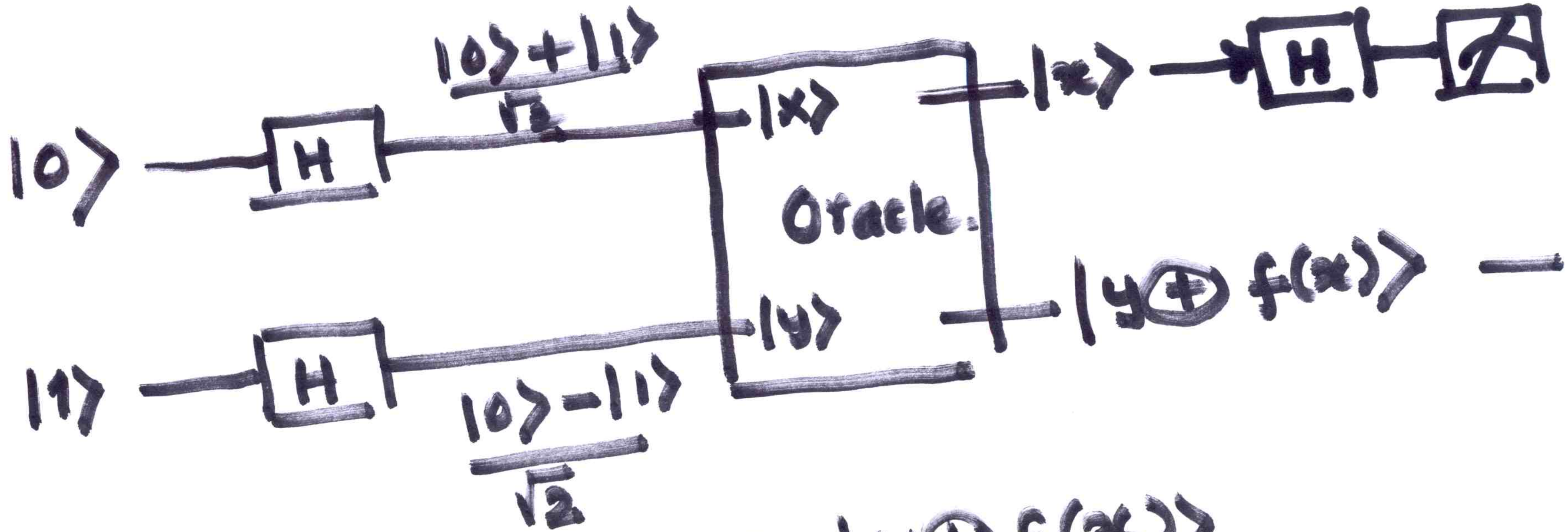
$$f(0) \neq f(1)$$

If $f(0) = 0$, $f(1) = 1$

$f(0) = 1$; $f(1) = 0$

Classical Computation.

2 Queries.



$$U_f |x\rangle \otimes |y\rangle \longrightarrow |x\rangle \otimes |y \oplus f(x)\rangle.$$

$$\longrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \left| \frac{|0\rangle - |1\rangle}{\sqrt{2}} \oplus f(x) \right\rangle$$

Input : $\frac{1}{2}\left[|00\rangle + |10\rangle - |01\rangle - |11\rangle\right]$

Oracle : $\frac{1}{2}\left[|0, 0\oplus f(0)\rangle + |1, 0\oplus f(1)\rangle - |0, \underline{1\oplus f(0)}\rangle - |1, 1\oplus f(1)\rangle\right]$

General $\longrightarrow$ $= \frac{1}{2}\left[|0, f(0)\rangle + |1, f(1)\rangle - |0, \overline{f(0)}\rangle - |1, \overline{f(1)}\rangle\right]$

If fn. is Constant $= \frac{1}{2}\left[(|0\rangle + |1\rangle)\big(\underbrace{f(0) - \overline{f(0)}}\big)\right].$

$$\cong \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$\xrightarrow[\text{1st Register}]{H} |0\rangle \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Function is balanced

$$\frac{1}{2}\Big[|0, f(0)\rangle + |1, f(1)\rangle - |0, f(1)\rangle - |1, f(0)\rangle\Big]$$

$$= \cancel{\pm}\, \frac{|0\rangle - |1\rangle}{\sqrt{2}} \otimes (\pm)\, \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$\Big\downarrow H$$

$$|1\rangle$$

Function const
1st Register $|0\rangle$

Function Balanced
1st Register $|1\rangle$

$$f : \{0,1\}^{\otimes n} \longmapsto \{0,1\}.$$

Function is $\begin{cases} \text{Constant} \\ \text{Balanced} \end{cases}$

$f(0\,0\,0) = 0$
$f(0\,0\,1) = 0$
$0\,1\,0 = 0$
$0\,1\,1 = 0$

$1\,0\,0$
$1\,0\,1$
$1\,1\,0$
$1\,1\,1$

Worst Case
$\frac{N}{2}+1$ trials.