Prof. D.K. Ghosh

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\{|e_j\rangle\}$$

$$|\psi_i\rangle = \sum_j c_{ij} |e_j\rangle$$

$$c_{ij} = \langle e_j|\psi_i\rangle$$

$$|\langle e_j|\psi_i\rangle|^2 = \text{Probability of finding } |\psi_i\rangle \text{ in } |e_j\rangle$$

$$\mathrm{Tr}\left[M_m^\dagger M_m \rho\right]$$

$$= \mathrm{Tr}\left[M_m^\dagger M_m \sum_i p_i |\psi_i\rangle\langle\psi_i|\right].$$

$$= \sum_i p_i \, \mathrm{Tr} \cdot \left(\underbrace{M_m^\dagger M_m |\psi_i\rangle}\langle\psi_i|\right)$$

$$= \sum_i p_i \langle\psi_i| M_m^\dagger M_m |\psi_i\rangle.$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

$$\sigma_Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$M_0 = |0\rangle\langle 0|$$

$$\rho = |\psi\rangle\langle\psi|$$

$$= (\alpha|0\rangle + \beta|1\rangle)(\alpha^* \overset{\langle 0|}{\cancel{|0\rangle}} + \beta^* \overset{\langle 1|}{\cancel{|1\rangle}})$$

$$Tr(M_0^\dagger M_0 \rho)$$

$$= Tr\left[|0\rangle \underbrace{\langle 0|0\rangle}\underbrace{\langle 0|[\alpha|0\rangle} + \beta|1\rangle][\alpha^*\langle 0| + \beta^*\langle 1|] \right]$$

$$= Tr\left[|0\rangle \cdot \alpha(\alpha^*\langle 0| + \beta^*\langle 1|) \right].$$

$$=$$

$$= |\alpha|^2 \langle 0|0\rangle = |\alpha|^2 .$$

$$\frac{M_0 \rho M_0^+}{tr(M_0^+ M_0 \rho)} = \frac{|0\rangle\langle 0| \, |\alpha|^2}{|\alpha|^2} = |0\rangle\langle 0|$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$\{0,1\}$ basis

$|0\rangle$ with $p = |\alpha|^2$

Next measurement in diagonal basis

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \quad : |+\rangle \quad \frac{1}{2}$$

$$0, + \quad : \quad \frac{|\alpha|^2}{2}$$

5(2)

$$\Psi = \alpha|0\rangle + \beta|1\rangle$$

$$= \frac{\alpha+\beta}{\sqrt{2}}|+\rangle + \frac{\alpha-\beta}{\sqrt{2}}|-\rangle$$

$$P(+) = \frac{|\alpha+\beta|^2}{2}$$

POVM

Positive Operators

$$\langle \psi | A \psi \rangle \geq 0 \quad \forall\, |\psi\rangle.$$

A collection $\{E_i\}$ of positive op.

$$\sum_i E_i = I.$$

$$E_i = M_i^\dagger M_i$$

$$p(i) = \langle \psi | E_i | \psi \rangle = \mathrm{Tr}\,[\rho E_i].$$

$$\rho \mapsto \rho' = \frac{M_i \rho M_i^\dagger}{\mathrm{Tr}\,(\rho E_i)}.$$

$$\rho \longmapsto \textstyle\sum_i M_i \rho M_i^\dagger \quad \text{if not read}$$

$$E_1 = \frac{\sqrt{2}}{\sqrt{2}+1} |1\rangle\langle 1|$$

$$E_2 = \frac{\sqrt{2}}{\sqrt{2}+1} \left[|0\rangle - |1\rangle \right] \left[\langle 0| - \langle 1| \right].$$

POVM

$$E_3 = I - E_1 - E_2.$$

$$\psi_1 = |0\rangle$$

$$\psi_2 = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$