$$\{M_m\} : m = 1, 2, \ldots n.$$

Probability of an outcome m

$$\langle \psi | M_m^\dagger M_m | \psi \rangle$$

Measurement operators.

$$\sum_{m=1}^{n} M_m^\dagger M_m = I$$

Post measurement state $\dfrac{M_m|\psi\rangle}{\sqrt{\langle \psi | M_m^\dagger M_m | \psi \rangle}}$

$$P_m^{\dagger} = P_m$$

$$P_m P_{m'} = P_m \delta_{mm'}.$$

$$p(m) = \langle \psi | P_m | \psi \rangle.$$

$$M = \sum_m m\, p^{(m)}. = \sum_m m \langle \psi | P_m | \psi \rangle$$

$$= \langle \psi | \underbrace{\sum_m m P_m} | \psi \rangle$$

$$= \langle \psi | M | \psi \rangle$$

$$|0\rangle, |1\rangle$$

$$M_0 = |0\rangle\langle 0|$$

$$M_1 = |1\rangle\langle 1|$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

$$p(0) = \langle\psi|0\rangle\langle 0|\psi\rangle$$

$$= (\alpha^*\langle 0| + \beta^*\langle 1|)|0\rangle\langle 0|(\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha^*\alpha = |\alpha|^2 \; ; \quad p(1) = |\beta|^2.$$

If result is $|0\rangle$

Post measurement state

$$\frac{M|\psi\rangle}{\sqrt{\langle\psi|M|\psi\rangle}} = \frac{|0\rangle\langle 0|(\alpha|0\rangle + \beta|1\rangle)}{|\alpha|}$$

$$= \frac{\alpha|0\rangle}{|\alpha|} = e^{i\varphi}|0\rangle$$

$\{|m\rangle\langle m|\}$

$\hookrightarrow$ basis states.

"measurement in a basis"

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$: Computational Basis

Diagonal Basis

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \qquad |1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \frac{\alpha+\beta}{\sqrt{2}}|+\rangle + \frac{\alpha-\beta}{\sqrt{2}}|-\rangle$$

$$\downarrow$$

$$p(+) = \frac{|\alpha+\beta|^2}{2}$$

$$p(-) = \frac{|\alpha-\beta|^2}{2}.$$

$$\sqrt{2}\,\frac{M_+|\psi\rangle}{|\alpha+\beta|} = e^{i\phi}|+\rangle$$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle.$$

$$M_0 = M_{00} + M_{01}$$

$$M_{01} \rightarrow |01\rangle\langle 01|$$

$$M_{00} \rightarrow |00\rangle\langle 00|$$

$$= |00\rangle\langle 00| + |01\rangle\langle 01|$$

1st qbit

2nd qubit

$$= \cancel{|00\rangle}\ \underbrace{|0\rangle\langle 0|}_{\text{1st Qubit}} \Big(\underbrace{|0\rangle\langle 0| + |1\rangle\langle 1|}_{\text{2nd Qubit}} \Big)$$

$$\Downarrow I$$

$$= |0\rangle\langle 0| \otimes I_2$$

$$\langle \psi | M_0 | \psi \rangle.$$

$$= \langle \psi | \left(|0\rangle\langle 0| \otimes I_2 \right) | \psi \rangle$$

$$= |\alpha|^2 + |\beta|^2.$$