

$$\{M_m\} : m = 1, 2, \dots, n.$$

Probability of an outcome m

$$\langle \psi | M_m^\dagger M_m | \psi \rangle$$

Measurement
 Operators.

$$\sum_{m=1}^n M_m^\dagger M_m = I$$

Post measurement state

$$\frac{M_m |\psi\rangle}{\sqrt{\langle \psi | M_m^\dagger M_m | \psi \rangle}}$$

$$P_m^\dagger = P_m$$

$$P_m P_{m'}^\dagger = P_m \delta_{mm'}$$

$$p(m) = \langle \psi | P_m | \psi \rangle$$

$$M = \sum_m m p(m) = \sum_m m \langle \psi | P_m | \psi \rangle$$

$$= \langle \psi | \underbrace{\sum_m m P_m} | \psi \rangle$$

$$= \langle \psi | M | \psi \rangle$$

$$|0\rangle, |1\rangle$$

$$M_0 = |0\rangle\langle 0|$$

$$M_1 = |1\rangle\langle 1|$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\begin{aligned} p(0) &= \langle \psi | 0 \rangle \langle 0 | \psi \rangle \\ &= (\alpha^* \langle 0 | + \beta^* \langle 1 |) | 0 \rangle \langle 0 | (\alpha | 0 \rangle + \beta | 1 \rangle) \\ &= \alpha^* \alpha = |\alpha|^2 ; \quad p(1) = |\beta|^2 \end{aligned}$$

If result is $|0\rangle$

Post measurement state:

$$\frac{M|\psi\rangle}{\sqrt{\langle\psi|M|\psi\rangle}} = \frac{|0\rangle\langle 0|(\alpha|0\rangle + \beta|1\rangle)}{|\alpha|}$$

$$= \frac{\alpha|0\rangle}{|\alpha|} = e^{i\phi}|0\rangle$$

$$\{|m\rangle\langle m|\}$$

↳ basis states.

"measurement in a basis"

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle : \text{Computational Basis}$$

Diagonal Basis

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$$

$$|1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \frac{\alpha + \beta}{\sqrt{2}} |+\rangle + \frac{\alpha - \beta}{\sqrt{2}} |-\rangle$$

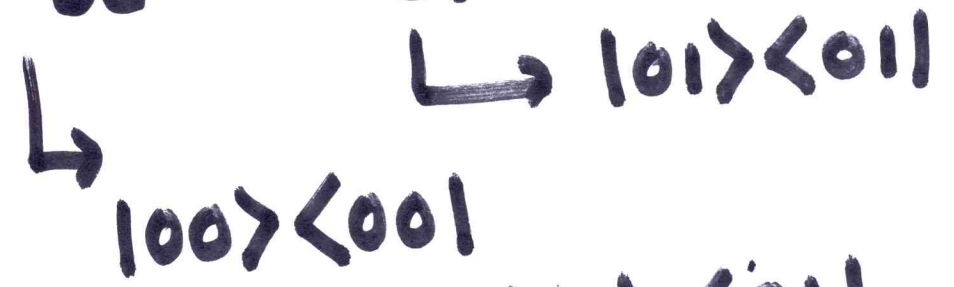
$$p(+)= \frac{|\alpha + \beta|^2}{2}$$

$$p(-)= \frac{|\alpha - \beta|^2}{2}$$

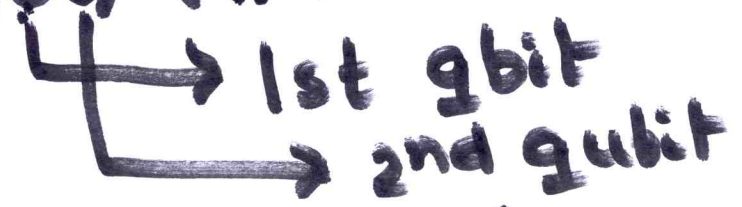
$$\frac{\sqrt{2} M_+ |\psi\rangle}{|\alpha + \beta|} = e^{i\phi} |+\rangle$$

$$|\psi\rangle = \alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle$$

$$M_0 = M_{00} + M_{01}$$



$$= |00\rangle\langle 00| + |01\rangle\langle 01|$$



$$= |00\rangle\langle 00| + |01\rangle\langle 01| \left(|0\rangle\langle 0| + |1\rangle\langle 1| \right)$$



$$= |0\rangle\langle 0| \otimes I_2$$

$$\begin{aligned} & \langle \psi | M_0 | \psi \rangle \\ &= \langle \psi | (|0\rangle\langle 0| \otimes I_2) | \psi \rangle \\ &= |\alpha|^2 + |\beta|^2. \end{aligned}$$