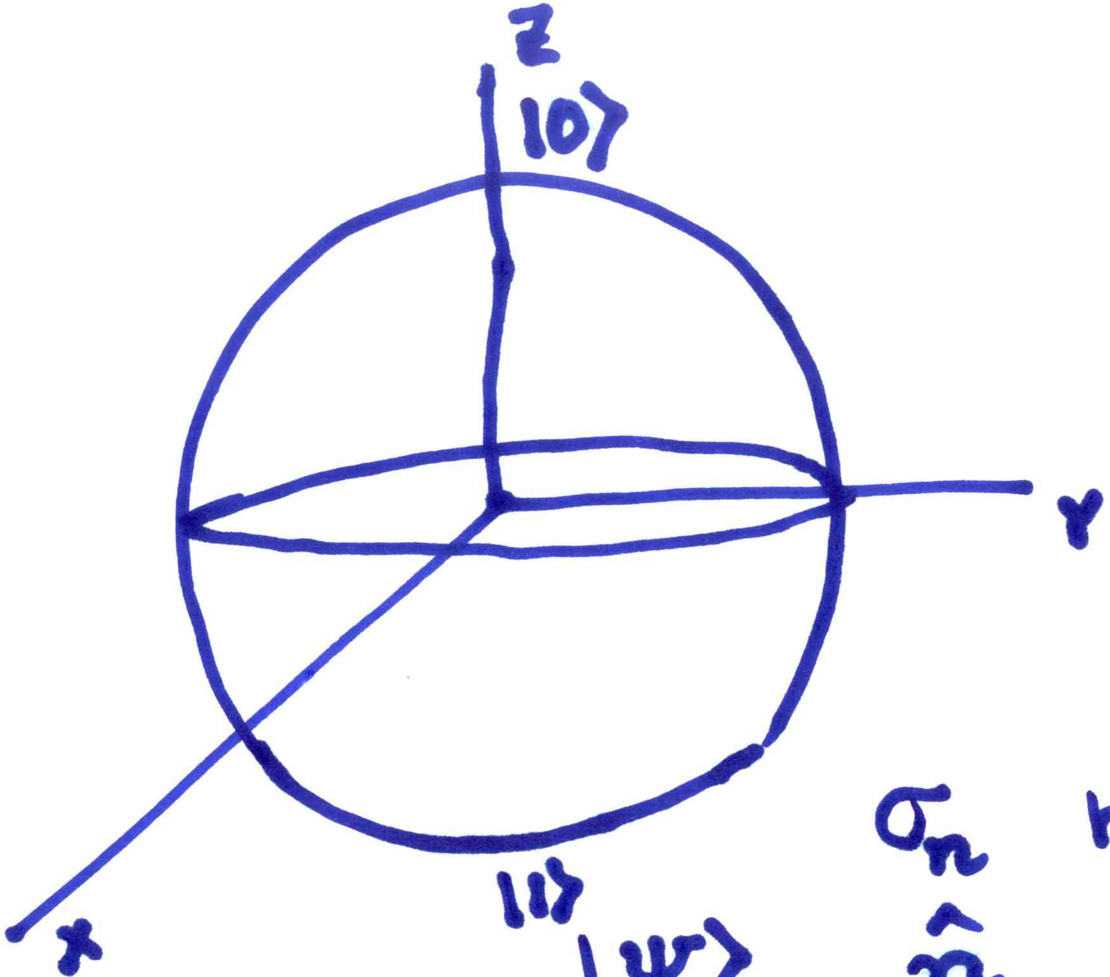
Prof. D.K. Ghosh
Lec 13
Date: 19/5/18



$|\psi\rangle$ $\hat{n}$

σ_n has an eigenvalue +1

$$|\psi\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\cdot\sin\frac{\theta}{2} \end{pmatrix}$$

$$\rho = |\psi\rangle\langle\psi|$$

$$= \frac{1}{2}\begin{pmatrix} 1+\cos\theta & e^{-i\varphi}\sin\theta \\ e^{+i\varphi}\sin\theta & 1-\cos\theta \end{pmatrix}$$

$$= \frac{I}{2} + \frac{1}{2}\sin\theta\cos\varphi\,\sigma_x$$

$$+ \frac{1}{2}\sin\theta\sin\varphi\cdot\sigma_y$$

$$+ \frac{\cos\theta}{2}\cdot\sigma_z .$$

$$= \frac{1}{2}\left[I + \hat{n}\cdot\vec{\sigma}\right]$$

$$\rho = \frac{1}{2}\left[I + \vec{a}\cdot\vec{\sigma} \right]; \quad |a| < 1$$

$$\cancel{z = \frac{1}{3}} \quad a_z = \frac{1}{3}$$

$$\rho = \frac{1}{2}\left[I + \frac{1}{3}\sigma_z \right].$$

$$= \frac{1}{2}\begin{pmatrix} 1+\frac{1}{3} & 0 \\ 0 & 1-\frac{1}{3} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 4/3 & 0 \\ 0 & 2/3 \end{pmatrix}$$

$$= \frac{2}{3}|0\rangle\langle 0| + \frac{1}{3}|1\rangle\langle 1|$$

$$\langle \sigma_x \rangle = \mathrm{Tr}(\sigma_x \rho)$$

$$= \frac{1}{2} \mathrm{Tr}\left(\sigma_x \left(I + a_x \sigma_x + a_y \sigma_y + a_z \sigma_z\right)\right)$$

$$= \frac{1}{2} \mathrm{Tr}\left[a_x\right] = a_x$$

$$\mathrm{Tr}\, \rho^2 = \mathrm{Tr}\left[\frac{1}{4}\left(I + \vec{a}\,\vec{\sigma}\right)^2\right]$$

$$= \frac{1}{4} \mathrm{Tr}\left[I^2 + a_x^2 \sigma_x^2 + a_y^2 \sigma_y^2 + a_z^2 \sigma_z^2 \right]$$

$$= \frac{1 + |a|^2}{2} \qquad \underline{|a| < 1}$$

$$< 1$$

$$\varrho^A = \mathrm{tr}_B(\varrho^{AB}).$$

$$\mathrm{Tr}_B\left[\underbrace{|a_1\rangle\langle a_2|}\otimes\underbrace{|b_1\rangle\langle b_2|}\right]$$

$$= |a_1\rangle\langle a_2| \; \mathrm{Tr}\underbrace{\left[|b_1\rangle\langle b_2|\right]}$$

$$\mathrm{Tr}\left[\underbrace{|b_1\rangle}\langle b_2|\right] = \sum_i \underbrace{\langle e_i|b_1\rangle}\underbrace{\langle b_2|e_i\rangle}$$

$$= \sum_i \langle b_2\underbrace{|e_i\rangle\langle e_i|}b_1\rangle$$

$$= \langle b_2|b_1\rangle$$

M = observable on A

$\tilde{M}$ = Measurement on AB.

$|m\rangle \otimes |\psi\rangle \rightarrow$ arbitrary state of B

$\uparrow$ eigen state of A

$$\tilde{M} = \sum_m m P_m \otimes I_B = M \otimes I_B$$

$$Tr[\rho_A M] = Tr[\rho_{AB} \tilde{M}]$$
$$= Tr[(M \otimes I_B)\rho_{AB}]$$
$$\rho_A = Tr_B(\rho_{AB})$$

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \rightarrow \text{Pure state.}$$

$$\rho = \frac{1}{2}\cdot\Big[\underbrace{|00\rangle\langle 00|} + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\Big]$$

$$\rho_1 = Tr_2\, \rho$$

$$= \frac{1}{2}\cdot\Big[\underbrace{|0\rangle\langle 0|}_{1st}\, Tr\big(\underbrace{|0\rangle\langle 0|}_{2nd}\big) + \cancel{|0\rangle\langle 1| \big(\langle 1|0\rangle\big)}$$

$$= \frac{1}{2}\Big[|0\rangle\langle 0| + |1\rangle\langle 1|\Big] \rightarrow \text{Mixed state.}$$