Prof. D. K. Ghosh
Lec. 12
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$|\psi\rangle \quad : \rho = |\psi\rangle\langle\psi|$

$\{|e_i\rangle\}$ is a basis

$$\langle A \rangle = \langle \psi | A | \psi \rangle$$

$$= \sum_i \sum_j c_i^* c_j \langle e_i | A | e_j \rangle .$$

$$\left| \begin{array}{l} c_j = \langle e_j | \psi \rangle \\ c_i^* = \langle \psi | e_i \rangle \end{array} \right.$$

$$\langle A \rangle = \sum_{ij} \langle \psi | e_i \rangle \langle e_j | \psi \rangle \langle e_i | A | e_j \rangle$$

$$\langle A \rangle = \sum_{ij} \underbrace{\langle e_j | \psi \rangle \langle \psi | e_i \rangle}_{\rho} A_{ij}$$

$$= \sum_{ij} \langle e_j | \rho \underbrace{| e_i \rangle \langle e_i |}_{} A | e_j \rangle$$

$$= \sum_{j} \langle e_j | \underline{\underline{\rho A}} | e_j \rangle$$

$$= \mathrm{Tr}(\rho A)$$

$A = I \qquad \boxed{\mathrm{Tr}\,\rho = 1}$

$\{e_i\}$

$\rho_{mn} = \langle e_m | \rho | e_n \rangle$

$\uparrow_{|\psi\rangle\langle\psi|}$

Diagonal elements

$\rho_{mm} = |\underbrace{\langle e_m | \psi \rangle}|^2 = |c_m|^2.$

Born probability of getting a state m.

$m \neq n$.

$$c_m = |c_m| e^{i\varphi_m}$$

$$c_n^* = |c_n| e^{-i\varphi_n}.$$

$$\rho_{mn} = \langle e_m | \psi \rangle \langle \psi | e_n \rangle$$

$$= c_m c_n^* \quad e^{i(\varphi_m - \varphi_n)}$$

$$= |c_m| |c_n| e$$

→ Interference terms.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

$$\rho = |\psi\rangle\langle\psi|$$

$$= \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix}\begin{pmatrix}1 & +i\end{pmatrix}.$$

$$= \frac{1}{2}\begin{pmatrix}1 & i\\-i & 1\end{pmatrix}$$

50% in $|0\rangle$

50% in $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$:

$$\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}\left[\frac{\langle 0| + \langle 1|}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]$$

$$= \frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1| + \frac{1}{4}|1\rangle\langle 0| + \frac{1}{4}|0\rangle\langle 1|$$

$$= \begin{pmatrix} 3/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}$$: Partially mixed

75%. $|0\rangle$

25%. $|1\rangle$

$$\rho = \begin{pmatrix} 3/4 & 0 \\ 0 & 1/4 \end{pmatrix}$$

$\rho_1 = \rho_2$ does not imply that two ensembles are the same

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

50% $|0\rangle$

50% $|1\rangle$

50% $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$

50% $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|+k\rangle + |-k\rangle\right]$$

$$P(x) = \mathrm{Tr}\left[S\,|x\rangle\langle x|\right]$$

$$= \langle x|S|x\rangle$$

$$= 1 + \cos(2kx)$$

Fringes

Incoherent Superposition.

$$S = \frac{1}{2}\left[|k\rangle\langle k| + |-k\rangle\langle -k|\right]$$

$P(x) = 1$ No Fringes.