$$|\psi\rangle = \alpha|x\rangle + \beta|y\rangle$$

$$|\phi\rangle = \gamma|x\rangle + \delta|y\rangle$$

Probability of state collapses to $|x\rangle$

$$p|\alpha|^2 + (1-p)|\gamma|^2.$$

Born Probability

Classical probability

$$|y\rangle : \quad p|\beta|^2 + (1-p)|\delta|^2.$$

$\mathcal{H}_A \otimes \mathcal{H}_B$

$|\Psi_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$ — No Entanglement

O_A an operator which measures some property of system A

$\langle \Psi_{AB} | O_A | \Psi_{AB} \rangle = \langle \Psi_A | O_A | \Psi_A \rangle$

$O_A \otimes I_B$ Pure state.

$$|\psi_{AB}\rangle = a|0\rangle_A \otimes |0\rangle_B + b|1\rangle_A \otimes |1\rangle_B.$$

$$(M_A \otimes I_B)$$

$$\langle\psi_{AB}| M_A \otimes I_B |\psi_{AB}\rangle \quad ?$$

$$\langle \Psi_{AB} \mid M_A \otimes I_B \mid \Psi_{AB} \rangle$$

$$= \left[a^* \langle 0_A | \underline{\underline{\langle 0_B |}} + b^* \langle 1_A | \langle 1_B | \right] \left| M_A \otimes \underline{\underline{I_B}} \right| \left[a |0\rangle_A |0\rangle_B + b |1\rangle_A |1\rangle_B \right]$$

$$= |a|^2 \langle 0_A | M_A | 0_A \rangle \overbrace{\langle 0_B | I_B | 0_B \rangle}$$

$$+ |b|^2 \langle 1_A | M_A | 1_A \rangle \underbrace{\langle 1_B | I_B | 1_B \rangle}$$

$$= |a|^2 \langle 0_A | M_A | 0_A \rangle + |b|^2 \langle 1_A | M_A | 1_A \rangle$$

Terms Dropped.

$$a^* b \langle 0_A | M_A | 1_A \rangle \underbrace{\langle 0_B | I_B | 1_B \rangle}_{=0}$$

Define.

$$\rho_A = |a|^2 \underbrace{|0_A\rangle\langle 0_A|}_{\text{operator.}} + |b|^2 |1_A\rangle\langle 1_A|$$

$$\Rightarrow \quad \mathrm{tr}[\rho_A M_A].$$

Density Operator (Matrix).

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$\langle \sigma_x \rangle = \frac{1}{2}\left[\left(\langle 0| + \langle 1|\right) \sigma_x \left(\underline{\underline{|0\rangle}} + \underline{\underline{|1\rangle}}\right)\right]$$

$$= \frac{1}{2}\left[\left(\langle 0| + \langle 1|\right)\left(|1\rangle + |0\rangle\right)\right]$$

$$= 1$$

Equal mixture of $|0\rangle$, $|1\rangle$.

$$\langle \sigma_x \rangle = \frac{1}{2}\langle 0|\underline{\sigma_x|0\rangle} + \frac{1}{2}\langle 1|\sigma_x|1\rangle$$

$$= 0$$

Density matrix

$$\frac{1}{2}\left(\underbrace{|0\rangle\langle 0| + |1\rangle\langle 1|}_{\substack{\text{completeness}\\ I}}\right) = \frac{I}{2}$$

$$\langle A\rangle = \mathrm{Tr}(\rho A)$$