Quantum No-Cloning Theorem

Cloning

$$|0\rangle \otimes |\psi\rangle \xrightarrow{U} |\psi\rangle \otimes |\psi\rangle$$

No unitary transformation exists which does it!

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If it were possible

$$\left.\begin{aligned} U|0\rangle \otimes |\psi\rangle &= |\psi\rangle \otimes |\psi\rangle \\ U|0\rangle \otimes |\phi\rangle &= |\phi\rangle \otimes |\phi\rangle \end{aligned}\right]$$

$$\begin{aligned} \langle \psi | \phi \rangle &= \overbrace{\langle \psi | \langle 0}\underbrace{|}\overbrace{0 \rangle | \phi \rangle} \\ &= \langle \psi, 0 | \underbrace{U^\dagger U}_{\to I} | 0, \phi \rangle \\ &= \langle \psi, \psi | \phi, \phi \rangle \\ &= |\langle \psi, \phi \rangle|^2 . \end{aligned}$$

Quantum Teleportation

Alice : $\alpha|0\rangle + \beta|1\rangle$: State 1

Alice and Bob share one qubit each of a Bell-states

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

bit 2 (Alice)

Bob. (Bit 3)

Alice : 1 & 2 Bob: 3

$$\frac{\alpha}{\sqrt{2}}\left[\underline{|0\rangle}\left(\underbrace{|00\rangle + |11\rangle}\right)\right] + \frac{\beta}{\sqrt{2}}\left[|1\rangle\left(|00\rangle + |11\rangle\right)\right]$$

$$= \frac{1}{\sqrt{2}}\cdot\left[\alpha\,|00\rangle|0\rangle + \alpha\,|01\rangle|1\rangle + \beta\,|10\rangle|0\rangle + \beta\,|11\rangle|1\rangle\right]$$

Alice's CNOT

$$= \frac{1}{\sqrt{2}}\left[\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|110\rangle + \beta\,|101\rangle\right]$$

Alice's H-Gate.

$$\frac{1}{\sqrt{2}}\Big[\alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|0\rangle|0\rangle$$

$$+\alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|1\rangle|1\rangle$$

$$+\beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|1\rangle|0\rangle$$

$$+\beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|0\rangle|1\rangle.\Big]$$

$$=\frac{1}{2}\Big[|00\rangle\left(\alpha|0\rangle+\beta|1\rangle\right)+|01\rangle\left(\alpha|1\rangle+\beta|0\rangle\right)$$

$$+|10\rangle\left(\alpha|0\rangle-\beta|1\rangle\right)+|11\rangle\left(\alpha|1\rangle-\beta|0\rangle\right)\Big]$$

Alice $|01\rangle$

Bob : $\alpha|1\rangle + \beta|0\rangle \xrightarrow{X} \alpha|0\rangle + \beta|1\rangle$

$= |\psi\rangle$

Alice $|11\rangle$

Bob $\underline{\alpha|1\rangle - \beta|0\rangle} \xrightarrow{X} \alpha|0\rangle - \beta|1\rangle$

$\xrightarrow{Z} \alpha|0\rangle + \beta|1\rangle.$