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Lec No. 06
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00, $\underline{\underline{01}}$, 10, 11

$|00\rangle = |0\rangle \otimes |0\rangle$

$|01\rangle$, $|10\rangle$, $|11\rangle$

$\underline{\alpha_{00}|00\rangle} + \underline{\alpha_{01}|01\rangle} + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$

Probability of measuring 0 in the 1st qubit

$$\frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{[|\alpha_{00}|^2 + |\alpha_{01}|^2]}}.$$

$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle.$

$$\psi_{+} = \frac{|01\rangle + |10\rangle}{\sqrt{2}}.$$

If 1st qubit = 1

$\therefore$ 2nd qubit = 0

Entanglement

Single Qubit Basis

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Basis for 2 Qubits.

$$|00\rangle = |0\rangle \otimes |0\rangle$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|10\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Computational Basis

$$\alpha|0\rangle + \beta|1\rangle$$

Unitary Operators

$$U^{\dagger}U = I.$$

<u>Reversible</u>

NOT Gate

$0 \longrightarrow 1$
$1 \longrightarrow 0$

Unitary operator.

Preserves "norm"

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Operators are 2x 2 matrices

$$U = e^{i\alpha} \exp\left[-i\underline{\underline{\theta}}\,\hat{n}\cdot\vec{\sigma}/2\right]$$

Pauli vectors $\sigma_x, \sigma_y, \sigma_z$

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$U^{\dagger}U =$$

A matrix can be written as a linear combination of $I, \sigma_x, \sigma_y, \sigma_z$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$UU^{\dagger}$$

NOT : $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$