$O \leftarrow$

$|\phi_1\rangle, |\phi_2\rangle, \dots |\phi_n\rangle. \leftarrow$ eigenvectors of O

$$\begin{matrix} \uparrow & \uparrow & & \uparrow \\ o_1 & o_2 & \dots & o_n \end{matrix}$$

$\rightarrow |\psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle + \dots \; c_n|\phi_n\rangle.$

$$E[O] = \langle\psi|O|\psi\rangle.$$

$$= \sum_{i=1}^{n} \underbrace{|c_i|^2}_{P(o_i)} o_i . \leftarrow \{|c_i|^2\}_{max}.$$

$\uparrow$

$a. \neq E[O]$

$$|\phi\rangle \doteq \begin{pmatrix} i \\ 1 \end{pmatrix}. \leftarrow$$

$$P_{|\phi\rangle} = \frac{|\phi\rangle\langle\phi|}{\langle\phi|\phi\rangle}.$$

$$= \frac{\begin{pmatrix} i \\ 1 \end{pmatrix}\begin{pmatrix} -i & 1 \end{pmatrix}}{-(i)^2+1}. = \frac{\begin{pmatrix} +1 & i \\ -i & 1 \end{pmatrix}}{2}.$$

7. $|\psi\rangle = \frac{1}{\sqrt{13}}\left(3|0\rangle - 2i\,|1\rangle\right)$. .. — 3.

$$|0\rangle = (|+\rangle + |-\rangle)/\sqrt{2}$$
$$|1\rangle = (|+\rangle - |-\rangle)/\sqrt{2}.$$

$$|\psi\rangle = \frac{1}{\sqrt{26}}\left((3-2i)\,|+\rangle + (3+2i)\,|-\rangle\right).$$

$$P(|+\rangle) = \left|\frac{3-2i}{\sqrt{26}}\right|^2 = \frac{1}{2}.$$

$$[x, p_x] = i\hbar$$

3.

$$U^{+} U = I.$$

$$U = \begin{pmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{pmatrix}$$

$$\rightarrow U^{+} U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U^{+} = \begin{pmatrix} \frac{1-i}{2} & \frac{1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{pmatrix}.$$

$$|\psi\rangle = \sum_i c_i |\phi_i\rangle.$$

basis states.

$$\langle\psi|\psi\rangle = 1.$$

$$|e_1\rangle, |e_2\rangle.$$

$$|\psi\rangle = c_1|e_1\rangle + c_2|e_2\rangle.$$

$$\langle\psi|\psi\rangle = (c_1^*\langle e_1| + c_2^*\langle e_2|)(c_1|e_1\rangle + c_2|e_2\rangle).$$

$$\langle e_1|e_2\rangle = 0.$$

$$= |c_1|^2 + |c_2|^2.$$

6. $|(\cos\theta_1 + \cos\theta_2)|^2 + |\sin\theta_1 + \sin\theta_2|^2 = 1.$

$$1 + 1 + 2\cos\theta_1\cos\theta_2 + 2\sin\theta_1\sin\theta_2$$

$$= 1.$$

$$\Rightarrow 2(\cos(\theta_1 - \theta_2)) = 1 - 2 = -1.$$

$$\cos(\theta_1 - \theta_2) = -1/2.$$