$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad : \quad \begin{array}{ll} +1 & \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -1 & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{array}$$

$$\sigma_n = \begin{pmatrix} \cos\theta & e^{-i\varphi} \sin\theta \\ e^{i\varphi} \sin\theta & -\cos\theta \end{pmatrix}$$

$$\sin\theta \cos\varphi - i \sin\theta \sin\varphi$$

$$= e^{-i\varphi} \sin\theta \qquad \lambda$$

Eigen values $\lambda = \pm 1$

Eigen state~~s~~ associated with $\lambda = +1$

$$|\theta,\varphi\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\cdot\sin\frac{\theta}{2} \end{pmatrix} \qquad \left.\begin{matrix}\theta = 0\\ \varphi = 0\end{matrix}\right|$$

Bloch - Sphere

$$|\theta,\varphi\rangle = \cos\frac{\theta}{2}\begin{pmatrix}1\\0\end{pmatrix} + e^{i\varphi}\cdot\sin\frac{\theta}{2}\begin{pmatrix}0\\1\end{pmatrix}$$

$$= \cos\frac{\theta}{2}\,|0\rangle + \underbrace{e^{i\varphi}\sin\frac{\theta}{2}\,|1\rangle}$$

North pole : $|0\rangle$

South pole : $\theta = \pi$, $\varphi = 0$ $|1\rangle$

Positive x-axis meets the equator : $\theta = \pi/2$, $\varphi = 0$

$$\frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]$$

Negative x-axis meets equator

$$\frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right]$$

$$\langle \sigma_z \rangle = \begin{pmatrix} \cos\frac{\theta}{2} & e^{i\varphi} \sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi} \cdot \sin\frac{\theta}{2} \end{pmatrix}$$

$$= \cos\theta$$

$$\alpha|0\rangle + \beta|1\rangle$$

$$\left.\begin{matrix} \dfrac{|\alpha|^2}{|\alpha|^2+|\beta|^2} \\ \dfrac{|\beta|^2}{|\alpha|^2+|\beta|^2} \end{matrix}\right]$$

Relative phase ?

$$|\psi\rangle = \left(|0\rangle + \underline{\underline{e}}^{i\theta}|1\rangle\right) \times \frac{1}{\sqrt{2}}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\theta}}{\sqrt{2}}|1\rangle.$$

Measure it in a diagonal basis.

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}} \quad ; \quad |1\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$$

$$|\psi\rangle = \frac{1}{2}\left[|+\rangle + |-\rangle\right] + \frac{e^{i\theta}}{2}\left[|+\rangle - |-\rangle\right].$$

$$= e^{i\theta/2}\left[\cos\frac{\theta}{2}|+\rangle - \sin\frac{\theta}{2}|-\rangle\right]$$