Prof. D. K. Ghosh
2-3- 6/5/16

$$|\alpha\rangle\langle\beta|$$

$$\begin{pmatrix} \\ \end{pmatrix}\begin{pmatrix} & \end{pmatrix}$$

$$A = \begin{pmatrix} 1+2i & -5i \\ 3i & 4 \end{pmatrix}$$

$$A^{+} = \begin{pmatrix} 1-2i & -3i \\ 5i & 4 \end{pmatrix}$$

$$B \cancel{A} = \begin{pmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{pmatrix} = B^{+}$$

$|\psi\rangle$ has a definite value λ for an observable

$$\hat{A}|\psi\rangle = \lambda|\psi\rangle$$

Eigenvalue

Eigenvector

$$P_m P_n = \delta_{mn} P_n$$

$$P_n^\dagger = P_n$$

$$\sum_n P_n = I$$

$\underline{\underline{\mathbb{C}^2}}$ $\quad \underline{\underline{e_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}}}$; $\quad e_2 \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \longleftarrow \lambda = -1$

$\lambda = +1$

Eigenstates of $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$.

Eigenvalues ± 1

$$P_1 = |e_1\rangle\langle e_1| = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1 & 1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

$$P_2 = |e_2\rangle\langle e_2| = \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\begin{pmatrix}1 & -1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = 1 \times \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + (-1) \cdot \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Wave Function Collapse.

$$|\Psi\rangle = \sum_n c_n |\underline{\underline{\Psi_n}}\rangle$$

$\{|\Psi_n\rangle\}$ Complete set of eigen states

$$\lambda_1, \quad \lambda_2, \quad \ldots \quad \lambda_n$$

$$\begin{array}{cc} \downarrow & \downarrow \\ |c_1|^2 & |c_2|^2 \end{array}$$

$$|c_n|^2 = \langle\Psi|P_n|\Psi\rangle$$

$$P_n = |\lambda_n\rangle\langle\lambda_n|$$

$$|\psi\rangle \mapsto P_n|\psi\rangle$$

Post-measurement state

$$\frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = c_1 \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$c_1 = \frac{3}{\sqrt{10}} \qquad c_2 = \frac{1}{\sqrt{10}}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Postulate 4

Time Evolution.

Schrödinger Equation

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle$$

$$|\psi_{(t)}\rangle = U(t) |\psi(0)\rangle$$

$$U(t) = e^{-iHt/\hbar}$$