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L-2 New
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normed vector space

inner product.

$\underline{\underline{|\psi\rangle}}$

$|\ \rangle$ Ket

Dual space

$\langle\psi|$

$\langle\ |$ bra

$\langle\ \ \rangle$

$\langle\psi|\phi\rangle$

$\langle\psi|\psi\rangle \geq 0$

Linear vector space

$$\langle \psi | a\phi_1 + b\phi_2 \rangle = a\langle \psi | \phi_1 \rangle + b\langle \psi | \phi_2 \rangle$$

$$\langle \psi | \phi \rangle^* = \langle \phi | \psi \rangle$$

$$\langle \psi | \psi \rangle = 1$$ Normalizatio

Ray

$c|\psi\rangle \quad |\psi\rangle$

Complex

$$\hat{A}\left[\alpha|\psi\rangle + \beta|\phi\rangle\right].$$
$$= \alpha\,\hat{A}|\psi\rangle + \beta\,\hat{A}|\phi\rangle$$

I = Identity

$$I|\psi\rangle = |\psi\rangle$$

$$\emptyset \quad [A, B] = AB - BA \neq I$$

$$AB = BA = I$$
$$B = A^{-1}$$

$$\langle \phi | \underline{\underline{A}} \Psi \rangle = \langle A^{\dagger} \phi | \Psi \rangle$$

$$(A^{\dagger})^{\dagger} = A$$

Hermitian Operators.

$$A^{\dagger} = A$$

$$\hat{A} = \underline{|\alpha\rangle\langle\beta|}$$

$$|\alpha\rangle \underbrace{\langle \beta | \Psi \rangle}_{c} = c\,|\alpha\rangle$$

$\mathbb{C}^n$

$\mathbb{C}^2$: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\mathbb{C}^3$: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$|\alpha\rangle\langle\beta|$$

$$\begin{pmatrix} \cdot\cdot \\ \vdots \\ \cdot\cdot \end{pmatrix}\begin{pmatrix} \cdots & \cdot\cdot \end{pmatrix}$$

$d \times d$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$= \begin{pmatrix} A_{11}B & A_{12}B \\ A_{21}B & A_{22}B \end{pmatrix} = \begin{pmatrix} A_{11}B_{11} & A_{11}B_{12} & \cdot\cdot \\ A_{11}B_{21} & A_{11}B_{22} & \cdot\cdot \\ \vdots & & \\ \cdot\cdot & & \end{pmatrix}$$