Self-assessment - Module 6

- A standard duality transformation of the xy model interchanges the roles of vortices with vorticity m and *m-fold anisotropies*, *i.e.* terms of the form

$$-\epsilon_m \int d^2r \cos(m\theta(\vec{r})) \tag{1}$$

, and sends the stiffness g to g^{-1} under this duality transformation. This transformation also interchanges the roles of the vortex fugacity $\tilde{\epsilon}_m$ for m-fold vortices and ϵ_m the coefficient of the m-fold anisotropy term.

You can read about this duality transformation in the famous paper of Jose *et. al.* (J. V. Jose, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, Phys. Rev. B **16**, 1217 (1977).). Using this, one can straightforwardly obtain the RG flow equations to leading order in both ϵ_p and the vortex fugacity $\tilde{\epsilon}$ assuming we have a fugacity only for single-vortices but various p-fold anisotropies:

$$\begin{aligned}
\frac{dg}{dl} &= \frac{\pi^2 p^2}{2}\epsilon_p^2 \exp(-\pi p^2/2g) - \frac{\pi^2\tilde{\epsilon}^2}{2} g^2 \exp(-\pi g/2) \\
\frac{d\epsilon_p}{dl} &= (2 - p^2/2g)\epsilon_p \\
\frac{d\tilde{\epsilon}}{dl} &= (2 - g/2)\tilde{\epsilon}\ ,
\end{aligned} \tag{2}$$

where a sum over the repeated label p is understood in first term of the first equation.

Show that these equations admits two *lines of fixed points with non-zero and continuously varying $\tilde{\epsilon}^{renorm}$*:

$$\begin{aligned}
g^{renorm} &= 4 \\
\epsilon_{p\neq 4}^{renorm} &= 0 \\
\epsilon_4^{renorm} &= \pm\tilde{\epsilon}^{renorm}
\end{aligned} \tag{3}$$

What does it say in physical terms about the finite-temperature transition to a four-fold symmetry breaking ordered state of an xy model with only four-fold anisotropy (and no other anisotropies)? You can read about the answer in the famous papers of Kadanoff (L. P. Kadanoff, Phys. Rev. Lett. **39**, 903 (1977); J. Phys. A **7**, 1399 (1978).)