## Introduction to Physics of Nanoparticles and Nano structures

1. Consider a monochromatic plane em-wave of the form $\vec{E}(z, t)=\vec{E}_{o} e^{i(q z-\omega t)}$, where

$$
\vec{E}_{o}=E_{\|} \hat{e}_{\|}+E_{\perp} \hat{e}_{\perp}, \quad \quad E_{\|}=a_{\|} \exp \left[-i \delta_{\|}\right], \quad \text { and } \quad E_{\perp}=a_{\perp} \exp \left[-i \delta_{\perp}\right]
$$

with $\hat{e}_{\|}$and $\hat{e}_{\perp}$ denoting orthogonal unit vectors on the wavefront, and $a_{\|}, a_{\perp}, \delta_{\|}$and $\delta_{\perp}$ having real values.
(a) Define the Stokes parameter $I, Q, U, V$ for the em-wave.
(b) Show that

$$
I=a_{\|}^{2}+a_{\perp}^{2}, \quad Q=a_{\|}^{2}-a_{\perp}^{2}, \quad U=2 a_{\|} a_{\perp} \cos \delta, \quad V=2 a_{\|} a_{\perp} \sin \delta,
$$

with $\delta=\delta_{\|}-\delta_{\perp}$.
(c) Show that $I^{2}=Q^{2}+U^{2}+V^{2}$.
(d) If the basis vectors $\hat{e}_{\|}$and $\hat{e}_{\perp}$ are rotated clockwise through an angle $\phi$ about an axis normal to the wavefront, then show that the new set of Stoke's parameter relative to the rotated vectors $\hat{e}_{\|}^{\prime}$ and $\hat{e}_{\perp}^{\prime}$ are given by

$$
I^{\prime}=I, \quad Q^{\prime}=Q \cos 2 \phi+U \sin 2 \phi, \quad U^{\prime}=-Q \sin 2 \phi+U \cos 2 \phi, \quad V^{\prime}=V .
$$

(e) If $a_{\perp}=a, a_{\|}=2 a$, and $\delta=\pi / 6$, what is the degree of the linear polarization and that of the circular polarization?
2. For a quasi-monochromatic wave prove the inequality $I^{2} \geq Q^{2}+U^{2}+V^{2}$.

