Part I: Physics of Nanoparticles

Questions on Module 4

1. Consider a monochromatic plane em-wave of the form $\vec{E}(z,t) = \vec{E}_o e^{i(qz-\omega t)}$, where

$$\vec{E}_o = E_{||}\hat{e}_{||} + E_{\perp}\hat{e}_{\perp}, \qquad \qquad E_{||} = a_{||}\exp[-i\delta_{||}], \qquad \qquad \text{and} \qquad E_{\perp} = a_{\perp}\exp[-i\delta_{\perp}].$$

with $\hat{e}_{||}$ and \hat{e}_{\perp} denoting orthogonal unit vectors on the wavefront, and $a_{||}$, a_{\perp} , $\delta_{||}$ and δ_{\perp} having real values.

- (a) Define the Stokes parameter I, Q, U, V for the em-wave.
- (b) Show that

$$I = a_{||}^2 + a_{\perp}^2, \qquad Q = a_{||}^2 - a_{\perp}^2, \qquad U = 2a_{||}a_{\perp}\cos\delta, \qquad V = 2a_{||}a_{\perp}\sin\delta,$$
 with $\delta = \delta_{||} - \delta_{\perp}$.

- (c) Show that $I^2 = Q^2 + U^2 + V^2$.
- (d) If the basis vectors \hat{e}_{\parallel} and \hat{e}_{\perp} are rotated clockwise through an angle ϕ about an axis normal to the wavefront, then show that the new set of Stoke's parameter relative to the rotated vectors \hat{e}'_{\parallel} and \hat{e}'_{\perp} are given by

$$I' = I,$$
 $Q' = Q \cos 2\phi + U \sin 2\phi,$ $U' = -Q \sin 2\phi + U \cos 2\phi,$ $V' = V.$

- (e) If $a_{\perp} = a$, $a_{||} = 2a$, and $\delta = \pi/6$, what is the degree of the linear polarization and that of the circular polarization?
- 2. For a quasi-monochromatic wave prove the inequality $I^2 \ge Q^2 + U^2 + V^2$.

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