Prof. D. Ghosh
Date :- 3/11/11
Lec. 35

1

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E}$$

$$\vec{J} = \sigma \vec{E}.$$

$E_x,\ H_y,\ \hat{z}$

$$(\vec{\nabla} \times \vec{E})_y + \mu \frac{\partial H_y}{\partial t} = 0$$

$$\boxed{\frac{\partial}{\partial z} E_x + \mu \frac{\partial H_y}{\partial t} = 0}$$

$$(\nabla \times H)_x = \epsilon \frac{\partial E_x}{\partial t} + \sigma E_x$$

$$-\frac{\partial}{\partial z} H_y = \epsilon \frac{\partial E_x}{\partial t} + \sigma E_x$$

$$\boxed{\frac{\partial}{\partial z} H_y + \epsilon \frac{\partial E_x}{\partial t} + \sigma E_x = 0}$$

$$E_x, H_y \sim e^{i\omega t}$$

$$\frac{\partial}{\partial t} \mapsto i\omega$$

$$\frac{\partial}{\partial z} E_x + i\omega\mu H_y = 0$$

$$\frac{\partial}{\partial z} H_y + (i\omega\epsilon + \sigma) E_x = 0$$

$$\frac{\partial^2}{\partial z^2} E_x + i\omega\mu(-)(i\omega\epsilon + \sigma) E_x = 0$$

$$\frac{\partial^2}{\partial z^2} E_x - \underbrace{i\omega\mu(i\omega\epsilon + \sigma)} E_x = 0$$

$$\frac{\partial^2}{\partial z^2} H_y - \underbrace{i\omega\mu(i\omega\epsilon + \sigma)}_{\gamma^2} H_y = 0$$

$$\frac{\partial^2}{\partial z^2} E_x - \gamma^2 E_x = 0$$

$$\gamma^2 = i\omega\mu(i\omega\epsilon + \sigma)$$

$$E_x = A \cosh \gamma z + B \sinh \gamma z$$

$$H_y = C \cosh \gamma z + D \sinh \gamma z$$

At $z = 0$ $\quad E_x = E_0 \; ; \; H_y = H_0$

$$A = E_0$$

$$C = H_0$$

$$\frac{\partial E_x}{\partial z} + i\omega\mu H_y = 0 \Leftarrow$$

$$A\gamma \sinh \gamma z + B\gamma \cosh \gamma z + i\omega\mu \left[C \cosh \gamma z + D \sinh \gamma z \right] = 0$$

$$A\gamma + i\omega\mu D = 0$$

$$B\gamma + i\omega\mu C = 0$$

$$D = -\frac{\gamma}{i\omega\mu} E_0$$

$$= -\frac{\sqrt{i\omega\mu(i\omega\epsilon+\sigma)}}{i\omega\mu} E_0$$

$$= -\sqrt{\frac{i\omega\epsilon+\sigma}{i\omega\mu}}\, E_0 = -\frac{E_0}{\eta}.$$

$$\eta = \sqrt{\frac{i\omega\mu}{\sigma + i\omega\epsilon}}.$$

$$B = -\eta H_0$$

$$E_x = E_0 \cosh \gamma z - \eta H_0 \sinh \gamma z$$
$$H_y = H_0 \cosh \gamma z - \frac{E_0}{\eta} \sinh \gamma z$$

$$z = -\ell$$

$$E_x = E_0 \cosh \gamma \ell + \eta H_0 \sinh \gamma \ell$$
$$H_y = H_0 \cosh \gamma \ell + \frac{E_0}{\eta} \sinh \gamma \ell$$

$l \gg$

$$\cosh \gamma l = \sinh \gamma l \approx \frac{e^{\gamma l}}{2}$$

$$E_x = \left(E_0 + \eta H_0\right)$$

$$H_y = \left(H_0 + \frac{E_0}{\eta}\right)$$

$$\frac{E_x}{H_y} = \frac{E_0 + \eta H_0}{H_0 + \frac{E_0}{\eta}} = \eta$$

Characteristic Impedence

$$\eta = \sqrt{\frac{i\omega\mu}{\sigma + i\omega\epsilon}}$$

$\sigma = 0$ $\quad$ $$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \times 10^{-7}}{9 \times 10^{-12}}} \approx 377\,\Omega$$

In vacuum.

$$\gamma = \sqrt{i\omega\mu(\sigma + i\omega\epsilon)}$$

$$= i\omega\sqrt{\mu\epsilon}.$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$= \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = -\nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

$$= -\mu \frac{\partial}{\partial t} \left[\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right].$$

$$-\nabla^2 E = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{E} = E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$E = E_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$k^2 = i\omega\mu\sigma + \mu\epsilon\omega^2$$

$$k = \sqrt{i\omega\mu\sigma + \mu\epsilon\omega^2}$$

$$= \sqrt{\omega\mu}\,\underline{(\omega\epsilon + i\sigma)}^{1/2} \longrightarrow \text{Re \& Im.}$$

$$= \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\left(1+\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}\right)^{1/2} + i\left(\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1\right)^{1/2}\right].$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[1+\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}\right]^{1/2}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1\right]^{1/2}$$

For a good conductor

$$\sigma \gg \omega\epsilon$$

$$\beta = \alpha \approx \omega\sqrt{\frac{\mu\epsilon}{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} = \sqrt{\frac{\omega\mu\sigma}{2}}.$$

$$v = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\sigma\mu}} \ll$$

$$E \sim e^{-\alpha z}$$

Skin depth

$$\boxed{\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\sigma\mu}}}$$

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$$

$\nu = 1\,MHz = 10^6$

Cu $\sim$ $\sigma \approx 6 \times 10^7\ \Omega^{-1}\, m^{-1}$

$$= \sqrt{\frac{2}{2\pi \times 10^6 \times 6 \times 10^7 \times 4\pi \times 10^{-7}}}$$

$= 0.067$ <u>mm</u>

Sea water 25 cm.

Fresh Water 7 m.

$$E_I + E_R = E_T$$

$$H_I + H_R = H_T$$

$$H_I = \frac{E_I}{\eta_1}$$

$$H_R = -\frac{E_R}{\eta_1}$$

η_1 medium 1

$$H_T = \frac{E_T}{\eta_2}$$

$$\frac{E_R}{E_I} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \frac{E_T}{E_I} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\eta_2 = \sqrt{\frac{i\omega\mu}{\sigma + i\omega\epsilon}}$$

$\sigma \gg \omega\epsilon$

$$\approx \sqrt{\frac{i\omega\mu}{\sigma}}$$

$\sqrt{i} = \frac{1+i}{\sqrt{2}}$.

$$= \frac{(1+i)}{\sqrt{2}} \sqrt{\frac{2\pi \times 10^6 \times 4\pi \times 10^{-7}}{6 \times 10^7}}.$$

$$= (1+i) \times 2.57 \times 10^{-4}$$

$$\eta_1 = 377\,\Omega$$

$$\frac{E_R}{E_I} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \approx -1$$ — Perfect Reflector

$$\frac{E_T}{E_I} = \frac{2\eta_2}{\eta_2+\eta_1} = \frac{5.14\times10^{-4}(1+i)}{377}$$

$$\approx 10^{-6}(1+i).$$

$$\frac{H_T}{H_I} \approx 2$$

$$Z_s = \frac{E_{\parallel}}{K_s}$$

$$J = J_0\, e^{-\gamma z}$$

$$K_s = \int_0^{\infty} J_0\, e^{-\gamma z}\, dz = \frac{J_0}{\gamma} = \frac{\sigma E_{\parallel}}{\gamma}$$

$$\gamma = \sqrt{i\omega\mu(\sigma + i\omega\epsilon)} \approx \sqrt{i\omega\mu\sigma} = \sqrt{\frac{\omega\mu\sigma}{2}}\,(1+i)$$

$$Z_s = \frac{E_{\parallel}}{K_s} = \frac{E_{\parallel}\cdot\gamma}{\sigma E_{\parallel}} = \frac{\gamma}{\sigma}.$$

$$Z_s = \frac{1}{\sigma}\sqrt{\frac{\omega\mu\sigma}{2}}\,(1+i) = \frac{1}{\sigma\delta}(1+i)$$

$$Z_s = \frac{1}{\sigma\delta}(1+i)$$

$$= R_s + iX_s.$$

$$\boxed{R_s = \frac{1}{\sigma\delta}}$$