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Lec. 31
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$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \frac{1}{c^2}\frac{1}{4\pi\epsilon_0}\nabla \int d^3x' \, \frac{\partial \rho(x',t)/\partial t}{|\vec{x}-\vec{x}'|} - \mu_0 \vec{J}$$

$$= -\frac{1}{c^2}\cdot\frac{1}{4\pi\epsilon_0}\nabla \int d^3x' \, \frac{\nabla'\cdot J}{|\vec{x}-\vec{x}'|} - \mu_0 \vec{J}$$

$$\vec{J} = \vec{J}_l + \vec{J}_t$$

$$\vec{\nabla}\cdot\vec{J}_t = 0 \qquad \vec{\nabla}\times\vec{J}_l = 0$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{J}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{J}) - \nabla^2 J$$

$$\nabla^2 J_t = -\nabla\times(\vec{\nabla}\times\vec{J}_t)$$

$$\nabla^2 J_l = \vec{\nabla}(\vec{\nabla}\cdot\vec{J}_l)$$

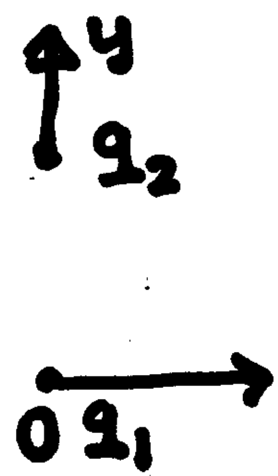
$$\int d^3x' \frac{\vec{\nabla}' \cdot \vec{J}}{|\vec{x}-\vec{x}'|} = -\int d^3x' \left(\vec{\nabla}' \cdot \vec{J}(x')\right) \nabla' \frac{1}{|x-x'|}$$

$$= -\int d^3x' \frac{\nabla'(\nabla' \cdot J(x')}{|x-x'|}$$

$$-\frac{\mu_0}{4\pi}\int d^3x' \, \frac{\nabla'(\nabla'\cdot J_\ell(x'))}{|x-x'|} - \mu_0 \vec{J}.$$

$$\nabla'(\nabla'\cdot J_\ell(x')) = \nabla'^2 J_\ell(x') + \nabla' \times (\nabla' \times J_\ell)$$
$$= \nabla'^2 J_\ell(x')$$

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{\mu_0}{4\pi}\int d^3x' \, \frac{\nabla'^2 J_\ell(x')}{|x-x'|} - \mu_0 \vec{J}$$
$$= -\frac{\mu_0}{4\pi}\int d^3x' \, J_\ell(x')\, \nabla^2 \frac{1}{|x-x'|} - \mu_0 J$$
$$= \mu_0 \vec{J}_\ell(\vec{x}) - \mu_0 \vec{J} = -\mu_0 \vec{J}_t$$


y
q2
0
q1

$$\vec{F} = \frac{\partial \vec{P}_{mech}}{\partial t} = \int \rho(\vec{E} + \vec{v}\times\vec{B})\, d^3x$$

$$= \int (\rho\vec{E} + \vec{J}\times\vec{B})\, d^3x$$

$$\nabla\cdot E = \frac{\rho}{\epsilon_0}$$

$$= \int \epsilon_0 \vec{E}(\nabla\cdot\vec{E})\, d^3x$$

$$+ \frac{1}{\mu_0}\int (\vec{\nabla}\times\vec{B})\times\vec{B}\, d^3x$$

$$- \epsilon_0 \int \frac{\partial\vec{E}}{\partial t}\times\vec{B}\, d^3x$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{J} = \frac{1}{\mu_0}\left[\vec{\nabla}\times\vec{B} - \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right]$$

$$= \frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\frac{\partial \vec{P}}{\partial t} = \int \epsilon_0 \vec{E}(\vec{\nabla}\cdot\vec{E})\, d^3x + \frac{1}{\mu_0}\int (\nabla\times B)\times B\, d^3x$$
$$- \epsilon_0 \int \frac{\partial \vec{E}}{\partial t}\times\vec{B}\; d^3x$$

$$\frac{\partial \vec{E}}{\partial t}\times\vec{B} = \frac{\partial}{\partial t}(\vec{E}\times\vec{B}) - \vec{E}\times\frac{\partial \vec{B}}{\partial t}$$
$$= \frac{\partial}{\partial t}(\vec{E}\times\vec{B}) + \vec{E}\times(\nabla\times\vec{E})$$

$$\frac{d\vec{P}}{dt} = \int \epsilon_0 \vec{E}(\nabla\cdot\vec{E}) - \frac{1}{\mu_0}\int B\times(\nabla\times B)\, d^3x$$
$$- \epsilon_0 \int \frac{\partial}{\partial t}(E\times B)\, d^3x - \epsilon_0 \int \vec{E}\times(\nabla\times E)\, d^3x$$
$$+ \frac{1}{\mu_0}\int \vec{B}(\nabla\cdot\vec{B})\, d^3x$$

$$\frac{d\vec{P}_{mech}}{dt} + \epsilon_0 \frac{d}{dt}\int (\vec{E}\times\vec{B})\, d^3x$$

$$= \frac{d\vec{P}_{mech}}{dt} + \frac{1}{c^2}\frac{d}{dt}\int \vec{S}\, d^3x$$

$$\left[\vec{E}(\nabla . E) - \vec{E}\times(\nabla\times\vec{E})\right]_x$$

$$= E_x\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) - E_y(\nabla\times E)_z + E_z(\nabla\times E)_y$$

$$= E_x\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) - \underline{E_y\left(\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\right)} + E_z\left(\frac{\partial}{\partial z}E_x - \underline{\frac{\partial}{\partial x}E_z}\right)$$

$$\frac{1}{2}\frac{\partial}{\partial x}E_x^2 - \frac{\partial}{\partial x}\frac{E_y^2}{2} - \frac{\partial}{\partial x}\frac{E_z^2}{2}$$

$$+ \frac{\partial}{\partial y}(E_x E_y) + \frac{\partial}{\partial z}(E_x E_z)$$

$$\frac{\partial}{\partial x}E_x^2 - \frac{\partial}{\partial x}\left(\frac{E_x^2 + E_y^2 + E_z^2}{2}\right) + \frac{\partial}{\partial y}(E_x E_y) + \frac{\partial}{\partial z}(E_x E_z)$$

$$\Downarrow$$

$$-\frac{1}{2}\frac{\partial}{\partial x}E^2$$