Dipan · Ghosh
a (17/10/11)

$$P(x) = A\,e^{-(x-\langle x\rangle)^2 B}$$

$$\mathcal{E} = -\frac{d}{dt}\oint \vec{B}\cdot\vec{dS}$$

$$= \oint \vec{E}\cdot\vec{dL}$$

$$= \int (\nabla\times\vec{E})\cdot\vec{dS}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{B}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\vec{l}_1 \times \vec{r}}{r^3}$$

$$\Phi_2 = \int_{S_2} \vec{B}_1 \cdot d\vec{S}_2$$

$$\Phi_2 \propto I_1 \Rightarrow \Phi_2 = M_{21} \cdot I_1$$

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M_{21} \cdot \frac{dI_1}{dt}$$

Mutual Inductance

$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{S}_2$$

$$= \int (\vec{\nabla} \times \vec{A}_1) \cdot d\vec{S}_2$$

$$= \oint \vec{A}_1 \cdot \vec{dl}_2$$

$$\vec{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{\vec{dl}_1}{r}$$

$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint\oint \frac{\vec{dl}_1 \cdot \vec{dl}_2}{r}.$$

$$\Rightarrow M_{21} = \frac{\mu_0}{4\pi} \oint\oint \frac{\vec{dl}_1 \cdot \vec{dl}_2}{r}$$

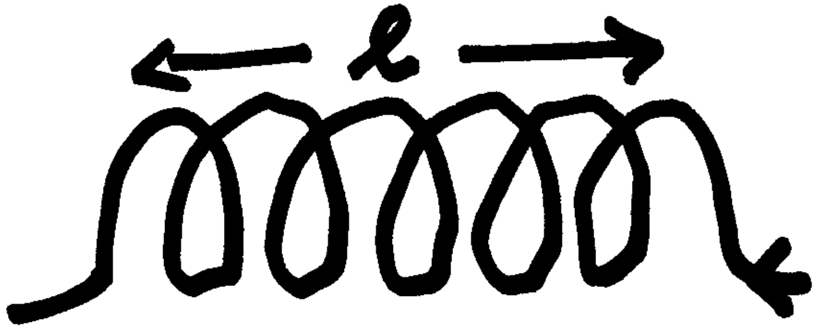
$$\equiv M_{12}$$

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{\partial \Phi}{\partial I}\cdot\frac{dI}{dt}$$

$$= -L\,\frac{dI}{dt}$$

$$\boxed{L = \frac{\partial \Phi}{\partial t}}$$

$$\vec{B} = \mu_0 n I \hat{z}$$



$N = n\ell$

Flux "linked" with each turn

$= \pi R^2 \mu_0 n I$

Total Flux linked

$= \pi R^2 \mu_0 n^2 \ell I.$

$$L = \frac{\partial \phi}{\partial I} = \pi R^2 \mu_0 n^2 \ell$$

$$\Phi_2 = (\mu_0 n_1 I)\,\pi R^2 n_2 \ell$$

$$M_{12} = \frac{\Phi_2}{I} = \mu_0 n_1 n_2 \pi R^2 \ell .$$

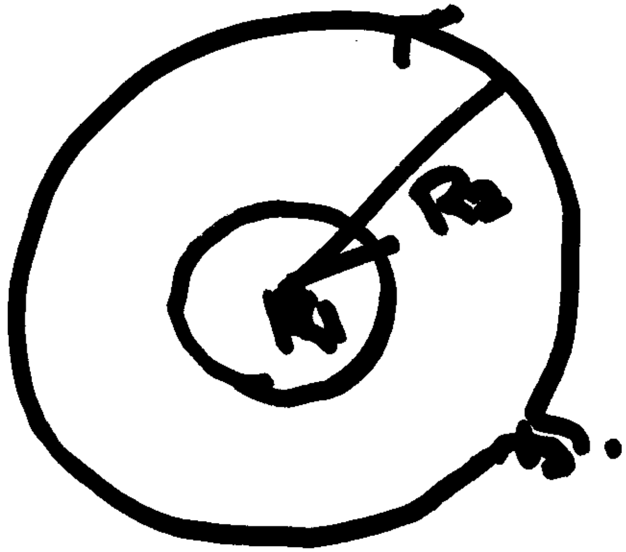
$$L_1 = \pi R^2 \mu_0 n_1^2 \ell$$

$$L_2 = \pi R^2 \mu_0 n_2^2 \cdot \ell$$

$$M_{12} = M_{21} = \sqrt{L_1 L_2}$$

$$M_{12} = k\sqrt{L_1 L_2}$$

$\hookrightarrow$ Coefficient of Coupling



$R_1 << R_2$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2R_2}\hat{k}$$

$$\Phi_1 = B_2 . \pi R_1^2$$
$$= \frac{\mu_0 I_2}{2R_2} . \pi R_1^2$$

$$M_{12} = \frac{\Phi_1}{I_2} = \pi R_1^2 \frac{\mu_0}{2R_2}$$

Flux change through i-th loop

$$\Phi_i = L_i I_i + \sum_{j\neq i} M_{ij} I_j$$

$$\mathcal{E}_i = -\frac{d\Phi_i}{dt} = L_i \frac{dI_i}{dt} + \sum_{j\neq i} M_{ij} \frac{dI_j}{dt}$$

Rate at which work is done

$$\mathcal{E}_i I_i = L_i I_i \frac{dI_i}{dt} + \sum_{j\neq i} M_{ij} I_i \frac{dI_j}{dt}$$

$$\sum_i \mathcal{E}_i I_i = \sum_i \frac{1}{2} L_i \frac{d}{dt} I_i^2 + \frac{1}{2} \sum_i \sum_{j\neq i} M_{ij} \frac{d}{dt}(I_i I_j)$$

$$W = \int \sum_i \mathcal{E}_i I_i \, dt$$

$$= \frac{1}{2} \sum_i L_i I_i^2 + \frac{1}{2} \sum_i \sum_{\substack{j \\ (i \neq j)}} M_{ij} I_i I_j$$

$$= \frac{1}{2} \sum_i I_i \Phi_i$$

$$= \frac{1}{2} \sum_i I_i \int \vec{B} \cdot d\vec{S}_i$$

$$= \frac{1}{2} \sum_i I_i \oint \vec{A} \cdot d\vec{l}_i$$

$$\rightarrow \frac{1}{2} \oint \vec{A} \cdot \vec{J} \, d^3 r$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \Rightarrow \nabla \cdot \vec{D} = \rho_{free}.$$

$$\nabla \cdot B = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{free}$$

$$\Phi_E = \int_{S2} \vec{D} \cdot d\vec{S}$$

$$\frac{\partial \Phi_E}{\partial t} = \frac{d}{dt} \int \vec{D} \cdot d\vec{S}$$

$$= \frac{d}{dt} \int \vec{\nabla} \cdot \vec{D} \, d^3r$$

$$= \frac{d}{dt} \int \rho \, d^3r$$

$$= \frac{dQ}{dt} = i_D$$

$$i_D = \frac{d\Phi_E}{dt}$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

$$\vec{\nabla} . \vec{J} + \frac{\partial}{\partial t} (\nabla . \vec{D}) = 0$$

$$\boxed{\vec{\nabla} . \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

Continuity.

$$A e^{-bx^2}$$: Guassian

$$\frac{1}{2\pi\sigma^2} e^{-\frac{(x-\langle x\rangle)^2}{2\sigma^2}}$$

$$A = \frac{1}{2\pi\sigma^2}$$

$$b = \frac{1}{2\sigma^2}$$

$$\langle x\rangle = 0$$