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Lec - 27

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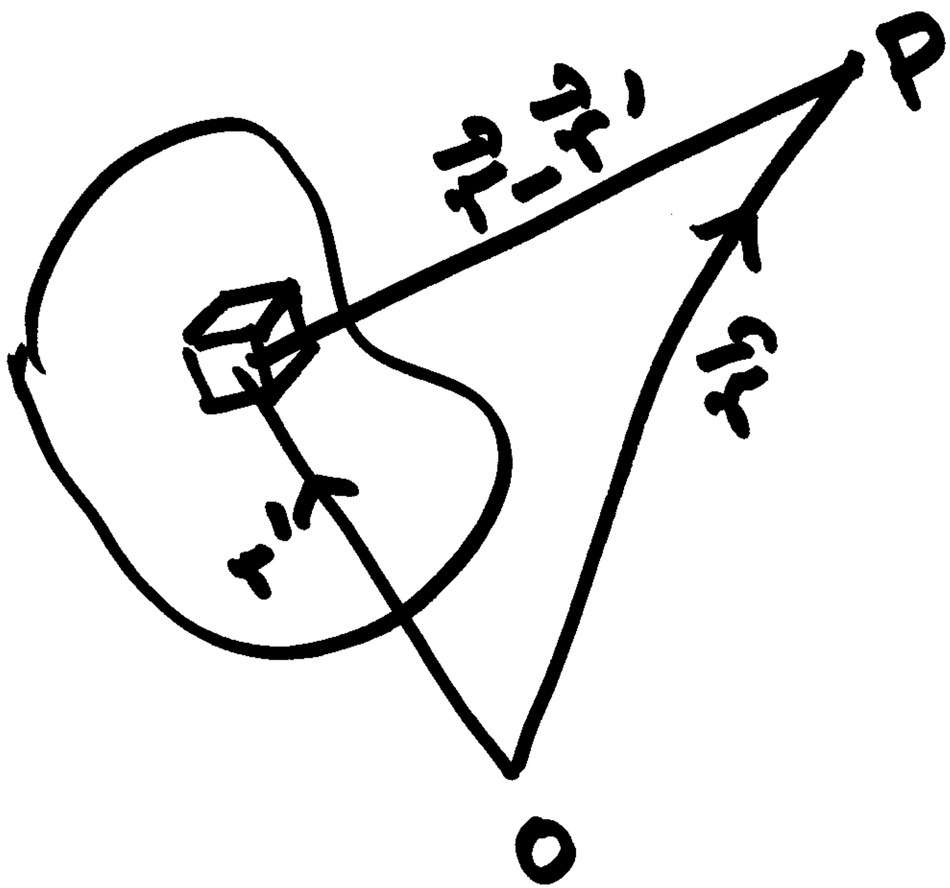
$$\phi_m = \frac{\vec{m}\cdot\vec{r}}{4\pi r^3}$$

$$\phi_m = -\frac{1}{4\pi}\int \frac{\nabla'\cdot\vec{M}(r')}{|\vec{r}-\vec{r}'|}d^3r' + \frac{1}{4\pi}\int\frac{\hat{n}\cdot\vec{M}(r')}{|\vec{r}-\vec{r}'|}ds$$

Magnetization

$$\lim_{\Delta v \to 0}\frac{\sum_i m_i}{\Delta v} \equiv \vec{M}$$


P
$\vec{r}-\vec{r}'$
$\vec{r}$
r'
O

$$\rho_{mb} = -\vec{\nabla} \cdot \vec{M}$$

$$\sigma_{mb} = \hat{n} \cdot \vec{M}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(r') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d^3r'$$

$$\vec{A}(\vec{r}) = -\frac{\mu_0}{4\pi}\int \frac{\hat{n}\cdot\vec{M}(r')}{|\vec{r}-r'|}ds' + \frac{\mu_0}{4\pi}\int \frac{\nabla' \times M(r')}{|\vec{r}-\vec{r}'|}d^3r$$

$$J_{mv}(\vec{r}) = \nabla \times M(\vec{r})$$

$$J_m^s(\vec{r}) = -\hat{n}\times\vec{M}(\vec{r})$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{M}(r') \times \nabla' \frac{1}{|r-r'|} d^3r'$$

$$\vec{B}(\vec{r}) = -\frac{\mu_0}{4\pi} . \nabla \times \int M(r') \times \nabla \frac{1}{|r-r'|} d^3r'$$

$$\nabla \times (\vec{A} \times \vec{C}) = \underline{\underline{\vec{A}(\vec{\nabla}.\vec{C})}} - \vec{C}(\vec{\nabla}.\vec{A}) + (\vec{C}.\vec{\nabla})\vec{A} - \underline{\underline{(\vec{A}.\vec{\nabla})\vec{C}}}$$

$$-4\pi\delta^3(\vec{r}-\vec{r}\,')$$

$$\vec{B}(\vec{r}) = -\frac{\mu_0}{4\pi}\int M(\vec{r}\,')\,\nabla^2\frac{1}{|\vec{r}-\vec{r}\,'|}\,d^3r'$$

$$+\frac{\mu_0}{4\pi}\int[\vec{M}(\vec{r}\,')\cdot\nabla]\,\nabla\frac{1}{|r-r'|}\,d^3r'$$

$$= \mu_0\vec{M}(\vec{r}) - \frac{\mu_0}{4\pi}\int(\vec{M}(r')\cdot\nabla_r)\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-r'|^3}\,d^3r$$

$$\nabla(\vec{A}\cdot\vec{C}) = (\vec{A}\cdot\vec{\nabla})\vec{C} + \vec{A}\times(\vec{\nabla}\times\vec{C}) + (\vec{C}\cdot\vec{\nabla})\vec{A} + \vec{C}\times(\nabla\times A)$$

$$-\frac{\mu_0}{4\pi}\int(\vec{M}(r')\cdot\nabla)\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}d^3r'$$

$$= -\frac{\mu_0}{4\pi}\nabla\int\vec{M}(r')\cdot\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-r'|^3}d^3r'$$

$$+\frac{\mu_0}{4\pi}\int\vec{M}(r')\times\left(\nabla\times\frac{\vec{r}-r'}{|r-r'|^3}\right)d^3r'$$

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J}_M + \vec{J}_c)$$

$$\vec{J}_M = \nabla \times \vec{M}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_M + I_c)$$

$$I_M = \int_S (\nabla \times M) \cdot ds = \oint \vec{M} \cdot dl$$

$$\oint (\vec{B} - \mu_0 \vec{M}) \cdot dl = \mu_0 I_c$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\oint \vec{H} \cdot \vec{dl} = I_c$$

$$\nabla \times \vec{H} = J_c$$

$$H = -\nabla \Phi_m$$

$$\vec{B} = \mu_0 \vec{H} \quad \text{if} \quad \vec{M} = 0$$

$\vec{M} = \chi_m \vec{H}$

$\chi_m \longrightarrow$ Magnetic Susceptibility

$\chi_m > 0$ — Paramagnet

$\chi_m < 0$ — Diamagnets.

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \underbrace{\mu_0(1+\chi_m)}\vec{H}$$
$$= \mu \vec{H}$$

$\mu \longrightarrow$ Permeability

$$\Phi_m(\vec{r}) = \frac{1}{4\pi}\int \frac{\hat{n}'\cdot\vec{M}(r')}{|\vec{r}-\vec{r}'|}\,ds'$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = 4\pi\sum_{l\,m}\frac{1}{2l+1}\frac{r_<^{\,l}}{r_>^{\,l+1}}\,Y_{lm}^{*}(\theta',\phi')\,Y_{lm}(\theta,\phi)$$

$$\hat{n}'\cdot M(r') = M\cos\theta'$$

$$= \sqrt{\frac{4\pi}{3}}\,M\,Y_{10}(\theta',\phi')$$

$$\int \cos\theta' \, Y^{*}_{\ell m}(\theta,\phi)\, d\Omega'$$

$$Y_{10}(\theta',\phi') \qquad : \ \ell = 1, \ m = 0$$

$$\Phi_m(\vec{r}) = \frac{M}{3} R^2 \sqrt{\frac{8\pi}{3}} \cdot Y_{10}(\theta,\phi) \cdot \frac{r_<^{\ell}}{r_>^{\ell+1}} .$$

$$= \frac{M}{3} \cdot R^2 \frac{r_<^{\ell}}{r_>^{\ell+1}} \cdot \cos\theta$$

$$= \frac{M}{3} R^2 \frac{r_<}{r_>^{2}} \cdot \cos\theta$$

Inside Sphere

$$\Phi_m = \frac{M}{3} r\cos\theta = \frac{M}{3} z.$$

Outside

$$\frac{M}{3}\frac{R^3}{r^2}\cos\theta$$

$$\vec{H} = -\nabla\Phi_m = -\frac{M}{3}$$

$$\vec{B} = \mu_0(\vec{H}+\vec{M}) = \frac{2}{3}\mu_0 M.$$

$$B = -\mu_0 \nabla \Phi_m$$

$$= -\mu_0 \left(\hat{r} \frac{\partial}{\partial r} + \hat{\theta} \cdot \frac{1}{r} \frac{\partial}{\partial \theta} \right) \cdot \frac{M}{3} \frac{R^3}{r^2} \cos\theta$$

$$= -\mu_0 \frac{MR^3}{3} \cdot \left[-\hat{r} \frac{2\cos\theta}{r^3} + \hat{\theta} \cdot \frac{\sin\theta}{r^3} \right].$$

$$= \mu_0 \frac{MR^3}{3} \left[3\, \vec{M} \cdot \vec{r} - \vec{M} \right]$$