$$d\vec{B}_1 = \frac{\mu_0}{4\pi} I_1 \frac{\vec{dl}_1 \times (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$\vec{B}_1 = \oint d\vec{B}_1$$

$$d\vec{F}_{21} = I_2 \vec{dl}_2 \times d\vec{B}_1$$

$$\vec{F}_{21} = \frac{\mu_0}{4\pi} I_1 I_2 \oint\oint \frac{\vec{dl}_2 \times (\vec{dl}_1 \times (\vec{r}_2 - \vec{r}_1))}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

$$\vec{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint\oint \frac{d\vec{l}_1 \times (d\vec{l}_2 \times (\vec{r}_1 - \vec{r}_2))}{|\vec{r}_2 - \vec{r}_1|^3}$$

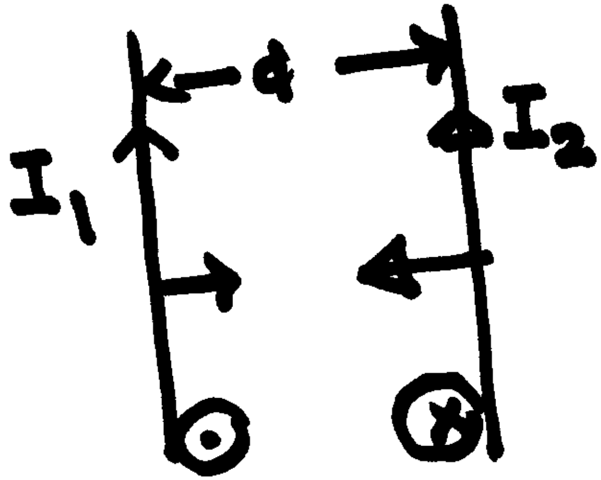
$$\frac{d\vec{l}_2 \times (d\vec{l}_1 \times (\vec{r}_2 - r_1))}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$= d\vec{l}_1 \left[ d\vec{l}_2 \cdot \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} \right] - \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} (d\vec{l}_2 \cdot d\vec{l}_1)$$

$$\oint\oint d\vec{l}_1 \left[ d\vec{l}_2 \cdot \frac{\vec{r}_2 - \vec{r}_1}{|r_2 - r_1|^3} \right] = -\oint\oint d\vec{l}_1 \left[ d\vec{l}_2 \cdot \nabla_2 \frac{1}{|\vec{r}_2 - \vec{r}_1|} \right]$$

$$\oint d\vec{l}_2 \cdot \nabla \frac{1}{|\vec{r}_2 - \vec{r}_1|} = \int_S \nabla \times \left( \nabla \frac{1}{|\vec{r}_2 - \vec{r}_1|} \right) ds$$

$$= 0$$




$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi d}(-\hat{k})$$

$$\vec{F}_{21} = I_2\, \hat{j} \times \vec{B}_1 .$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d}\, \hat{j} \times (-\hat{k})$$

$$= + \frac{\mu_0 I_1 I_2}{2\pi d}(-\hat{i})$$

Parallel currents <u>attract</u>

**Frame S**: Force is Electric

$$F = \frac{\lambda_1 \lambda_2 L \,\hat{r}}{2\pi\epsilon_0 d}$$

**Frame S'**

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \; ; \quad L' \mapsto L/\gamma$$

$$\lambda' = \lambda\gamma$$

$$F' = \frac{1}{2\pi\epsilon_0} \cdot \frac{1}{d} (\gamma\lambda_1)(\gamma\lambda_2)\frac{L}{\gamma}$$

$$= \gamma F$$

(Calculated by S')

$$F' = \gamma F + F'_m = \frac{F}{\gamma}$$

$$F'_m = \frac{F}{\gamma} - \gamma F = \gamma F\left(\frac{1}{\gamma^2} - 1\right)$$

$$= -\underline{\underline{\gamma F}}\,\frac{v^2}{c^2}.$$

$$= -F'\,\frac{v^2}{c^2}.$$

$$F'_m = -\frac{1}{2\pi\epsilon_0}\,\frac{\lambda'_1\lambda'_2 L'}{d}\cdot\frac{v^2}{c^2} \qquad (I = \lambda v)$$

$$= -\,\frac{I_1 I_2 L'}{2\pi\epsilon_0 c^2 d}.$$

$$\boxed{\mu_0\epsilon_0 = \frac{1}{c^2}} \qquad \mu_0 \to \frac{1}{\epsilon_0 c^2}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}$$

$$\vec{B} = \text{Curl}\,\vec{A} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\vec{B} = \nabla\times(\nabla\times A)$$

$$= \nabla(\vec{\nabla}\cdot\vec{A}) - \nabla^2 A$$

$$= \mu_0 \vec{J}.$$

In Coulomb Gauge

$$\nabla^2 A = -\mu_0 \vec{J}$$

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0}$$

$$\vec{A} \mapsto \vec{A} + \nabla\psi$$

$$\vec{\nabla}\cdot\vec{A} \Rightarrow \vec{\nabla}\cdot\vec{A} + \nabla^2\psi$$

$$\nabla^2\psi = \vec{\nabla}\cdot\vec{A}$$

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$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \vec{\nabla} \times \int \frac{\vec{J}(r')}{|\vec{r} - \vec{r}'|} d^3 r'.$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r'.$$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{|\vec{r} - \vec{r}'|} d^3 r'$$