$$-\epsilon_0 \frac{\partial \phi_1}{\partial r}\bigg|_{r=a} = -\epsilon \frac{\partial \phi_2}{\partial r}.$$

$$-\epsilon_0 A_1 + \epsilon_0 \frac{2B_1}{a^3} = -\epsilon A_2$$

$$A_2 = -\frac{\epsilon_0}{\epsilon} E_0 - \frac{\epsilon_0}{\epsilon} \frac{2B_1}{a^3}$$

$$A_2 = -E_0 + \frac{B_1}{a^3}$$

$$B_1 = E_0 a^3 \cdot \frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} = E_0 a^3 \frac{x-1}{x+2}$$

$$A_2 = -\frac{3E_0}{x+2}; \quad \phi_2 = \frac{-3E_0}{x+2} r\cos\theta$$

$$E_0 - E = E_0 - \frac{3E_0}{\kappa+2} = \frac{\kappa-1}{\kappa+2} E_0$$

$$\frac{P}{4\pi\epsilon_0 a^3}$$

$$\vec{P} = 4\pi\epsilon_0 a^3 \cdot \frac{\kappa-1}{\kappa+2} E_0 \hat{z}$$

$$\vec{P} = \frac{\vec{P}}{\frac{4\pi}{3} a^3} = 3\epsilon_0 \cdot \frac{\kappa-1}{\kappa+2} E_0 \hat{z}$$

$$\vec{E} = -\frac{\kappa-1}{\kappa+2} E_0 = -\frac{\vec{P}}{3\epsilon_0} \hat{z} = -\frac{P}{3\epsilon_0}.$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

$$= -\frac{K-1}{K}\cdot\frac{q}{4\pi}\,\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right)$$

$$\nabla\left(\frac{1}{r^3}\right)\cdot\vec{r} + \frac{1}{r^3}\,\vec{\nabla}\cdot\vec{r}$$

$$= -\frac{3\hat{r}}{r^4}\cdot\vec{r} + \frac{3}{r^3} = 0$$

$$\sigma_b = \vec{P} \cdot \hat{n} = -\vec{P} \cdot \hat{r}$$

$$= -\frac{x-1}{x} \cdot \frac{q}{4\pi r^2}$$

$$Q_b = -\frac{x-1}{x} q.$$

$$q_{eff} = q + Q_b = \frac{q}{x}$$

$$W = \frac{1}{2}\int \rho(\vec{x})\,\Phi(\vec{x})\,d^3x$$

$$\rho \mapsto \rho + \delta\rho$$

$$\vec{\nabla}\cdot\vec{D} = \rho$$

$$\delta\rho = \vec{\nabla}\cdot(\delta\vec{D})$$

$$\delta W = \int \vec{E}\cdot\delta\vec{D}\,d^3x$$

$$\delta W = \int \delta\rho\,\phi(x)\,d^3x$$

$$= \int \vec{\nabla}\cdot(\delta\vec{D})\,\phi(x)\,d^3x$$

$$\delta W = \cancel{\phi(x)(\delta \vec{D})\Big|} + \int \delta\vec{D}\cdot\vec{E}\, d^3x$$

$$= \int \underline{\underline{\delta\vec{D}}}\cdot\vec{E}\, d^3x$$

$$W = \int d^3x \int_0^{\vec{D}} \vec{E}\cdot\delta\vec{D}$$

$$\vec{E}\cdot\delta\vec{D} = \frac{1}{2}\delta(\vec{E}\cdot\vec{D})$$

$$\boxed{W = \frac{1}{2}\int \vec{E}\cdot\vec{D}\, d^3x}$$

Hysteresis

$$E_0 - E = E_0 - \frac{3E_0}{x+2}$$

$$= \frac{x-1}{x+2} E_0$$