$$\frac{1}{|\vec{r}-\vec{r}\,'|} = \frac{1}{\left[r^2 + r'^2 - 2rr'\cos\theta\right]^{1/2}}$$

$$= \frac{1}{r}\left[1 \underbrace{- \frac{2r'}{r}\cos\theta + \left(\frac{r'}{r}\right)^2}\right]^{-1/2}$$

$$= \frac{1}{r}\left[1 - \frac{1}{2}\left\{-\frac{2r'}{r}\cos\theta + \left(\frac{r'}{r}\right)^2\right]\right.$$

$$\left. + \frac{3}{8}\left[-\frac{2r'}{r}\cos\theta + \left(\frac{r'}{r}\right)^2\right]^2 + \cdots\right].$$

$$= \frac{1}{r} + \frac{r'}{r^2}\cos\theta + \frac{r'^2}{2r^3}\left(3\cos^2\theta - 1\right) + \cdots$$

$$(\vec{r}\cdot\vec{r}\,')^2 = (xx' + yy' + zz')^2 .$$

$$\vec{r} = (x_1, x_2, x_3)$$
$$\vec{r}\,' = (x_1', x_2', x_3')$$

$$= \sum_{i=1}^{3}\sum_{j=1}^{3} x_i x_i' x_j x_j' .$$

$$r^2 = \sum_{i=1}^{3}\sum_{j=1}^{3} x_i x_j \delta_{ij}$$

$$\delta_{ij} = 1 \quad \text{if} \quad i = j$$
$$= 0$$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_{\text{vol}} \rho(r')\cdot\frac{1}{|\vec{r}-\vec{r}\,'|}\, d^3r'$$

$$= \frac{1}{4\pi\epsilon_0}\Big[\frac{1}{r}\int_{\text{vol}}\rho(r')\,d^3r' + \frac{\vec{r}}{r^3}\cdot\int_{\text{vol}}\vec{r}\,'\rho(r')\,d^3r'$$

$$+\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{x_i x_j}{2r^5}\int_{\text{vol}}(3x_i' x_j' - \delta_{ij} r'^2)\rho(r')\,d^3r' + \cdots\Big]$$

$$\frac{\vec{r}}{r^3}\cdot\underbrace{\boxed{\int \vec{r}\,'\rho(r')\,d^3r'}}_{=\ \vec{p}}$$

$\Downarrow$ dipole moment.

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\,\frac{\vec{r}\cdot\vec{p}}{r^3}.$$ term.

$$q_{l\underline{\underline{m}}} = \int Y_{lm}^{*}(\theta', \varphi')\, r'^{l} \rho(r')\, d^3 r'$$

$$q_{00} = \int \frac{1}{\sqrt{4\pi}} \rho(r')\, d^3 r' = \frac{Q}{\sqrt{4\pi}}$$

$$q_{11} = \int -\sqrt{\frac{3}{8\pi}} \sin\theta'\, e^{-i\varphi'} \rho(r')\, r'\, d^3 r'.$$

$$= -\sqrt{\frac{3}{8\pi}} \int \rho(r') \sin\theta' \left[\cos\varphi' - i \sin\varphi'\right] r'\, d^3 r'$$

$$= -\sqrt{\frac{3}{8\pi}} \int (x' - i y') \rho(r')\, d^3 r'$$

$$= -\sqrt{\frac{3}{8\pi}} \,(p_x - i p_y)$$

$$q_{10} = \sqrt{\frac{3}{4\pi}} \int z' \rho(r')\, d^3 r' = \sqrt{\frac{3}{4\pi}}\, p_z$$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{P}(\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,d^3r'$$

$$= \frac{1}{4\pi\epsilon_0}\int \vec{P}(\vec{r}\,')\cdot\nabla'\frac{1}{|\vec{r}-\vec{r}\,'|}\,d^3r'$$

$$\nabla\frac{1}{r} = -\frac{\hat{r}}{r^2}$$

$$\nabla\cdot\left(f(r)\vec{V}\right)$$

$$= f(r)\,\nabla\cdot\vec{V} + \nabla f(r)\cdot\vec{V}\ .$$