$$\phi(r) = -\frac{\lambda}{2\pi\epsilon_0}\ln r \qquad E = \frac{\lambda}{2\pi\epsilon_0 r}.$$

$$\phi(r) = -\frac{\lambda}{2\pi\epsilon_0}\ln r_1 - \frac{\lambda'}{2\pi\epsilon_0}\ln r_2.$$

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$

$$r_2^2 = r^2 + b^2 - 2br\cos\theta.$$

$$\Phi(r) = -\frac{1}{4\pi\epsilon_0}\Big[\lambda\ln(r^2+a^2-2ar\cos\theta) + \lambda'\ln(r^2+b^2-2br\cos\theta)\Big].$$

$$\left.\frac{\partial \phi}{\partial \theta}\right|_{r=R} = 0$$

$$b\left(a^2 + R^2\right) = a\left(b^2 + R^2\right).$$

$$\boxed{b = R^2/a}$$

$$\vec{E} = -\nabla \Phi$$

$$\vec{E} = \vec{\nabla} \times \vec{A} \qquad \vec{A} = A\,\hat{k}$$

$$E_x = -\frac{\partial \phi}{\partial x} = (\vec{\nabla} \times \vec{A})_x = \frac{\partial}{\partial y} A.$$

$$E_y = -\frac{\partial \phi}{\partial y} = (\vec{\nabla} \times \vec{A})_y = -\frac{\partial A}{\partial x}$$

$$\boxed{\begin{aligned} E_x &= -\frac{\partial \phi}{\partial x} = +\frac{\partial A}{\partial y} \\ E_y &= -\frac{\partial \phi}{\partial y} = -\frac{\partial A}{\partial x} \end{aligned}}$$

$x, y \qquad z = x + iy$

$$f(z) = u(x,y) + i\,v(x,y).$$

$$z \longrightarrow f(z) = \omega.$$

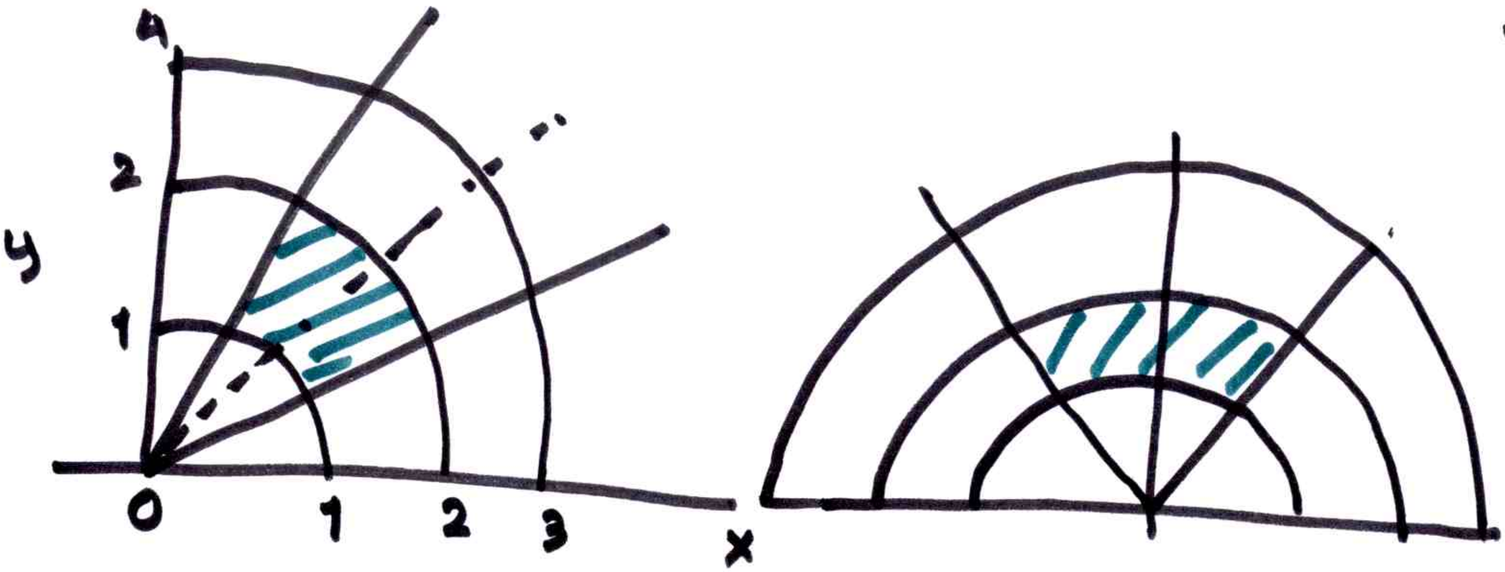
~~$u = x +$~~ $\boxed{f(z) = z^2}$

$$f(z) = (x+iy)^2$$

$$= \underbrace{x^2 - y^2} + \underline{2i\,xy}.$$

$u = x^2 - y^2$

$v = 2xy$

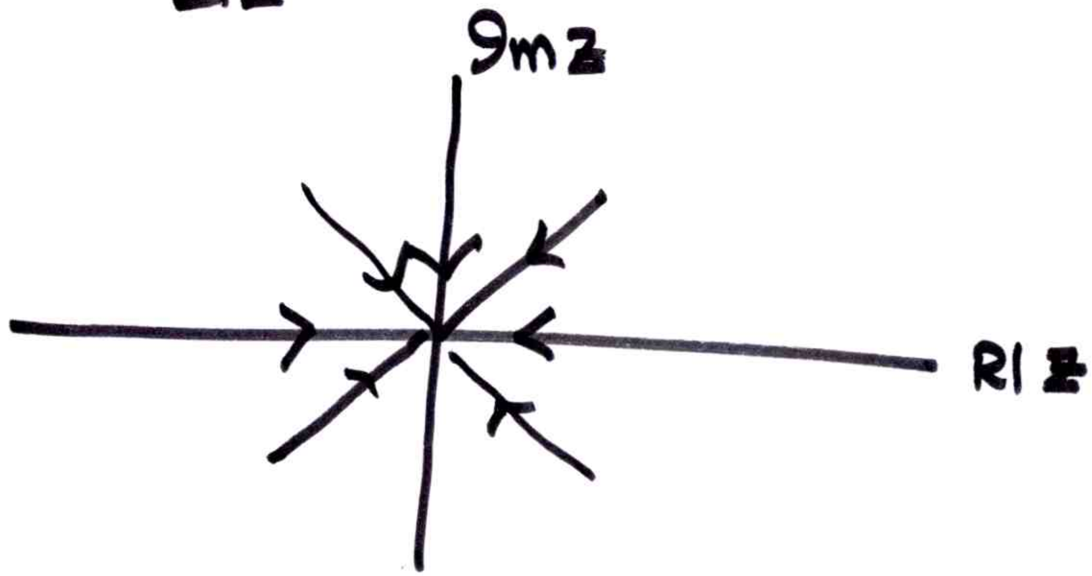


$$f(z) = z^2 .$$

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

f : function of real var.

$$\frac{df}{dz} = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$



$$\boxed{\begin{aligned}\frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y} &= -\frac{\partial v}{\partial x}\end{aligned}}$$

Cauchy - Riemann Conditions.

Analytic Functions.

$$E_x = -\frac{\partial \phi}{\partial x} = \frac{\partial A}{\partial y}$$

$$E_y = -\frac{\partial \phi}{\partial y} = -\frac{\partial A}{\partial x}.$$

$$(u, v) \longmapsto (\Phi, -A)$$

$$w = \Phi - iA$$

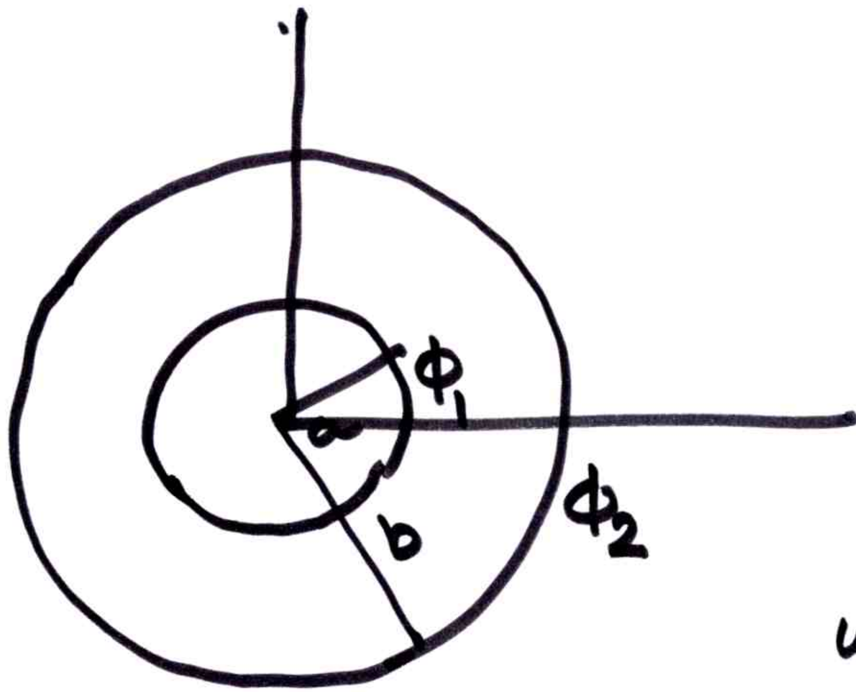
$$\frac{\partial w}{\partial x} = \frac{\partial \Phi}{\partial x} - i\frac{\partial A}{\partial x} = -E_x + iE_y.$$

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{-\partial A/\partial x}{\partial A/\partial y}.$$

$$\frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}\cdot dy = dA = 0$$

$$A = \text{Constant}$$

$$u = \Phi$$



$$a < r < b.$$

$$\boxed{\begin{aligned} A &= \text{Constant} \\ C &= \end{aligned}}$$

$$\omega = A \ln z + C$$
$$= u + iv.$$

$$\ln z = \ln r + i\theta$$
$$r = \sqrt{x^2+y^2}$$
$$\tan\theta = \frac{y}{x}.$$

$$u = A \ln r + C$$
$$v = A\theta$$

$$u = \phi$$

$$\phi = A \ln r + C.$$

$$\phi_1 = A \ln a + C \leftarrow$$

$$\phi_2 = A \ln b + C. \leftarrow$$

$$\phi_2 - \phi_1 = A \ln \frac{b}{a}$$

$$A = \frac{\phi_2 - \phi_1}{\ln(b/a)}. \checkmark$$

$$C = \phi_1 - A \ln a$$

$$= \frac{\phi_2 \ln a - \phi_1 \ln b}{\ln a - \ln b}. \checkmark$$

$$\phi = \frac{\phi_2 - \phi_1}{\ln(b/a)} \cdot \ln r + \frac{\phi_2 \ln a - \phi_1 \ln b}{\ln(a/b)}$$

$$E = -\frac{\partial \phi}{\partial r} = -\frac{A}{r}\hat{r} \; ; \quad \sigma = -\epsilon_0 \frac{A}{a}$$

$$Q_{in} = 2\pi a \sigma .$$

$$A = -\frac{a\sigma}{\epsilon_0} = -\frac{Q_{in}}{2\pi\epsilon_0} .$$

$$\phi = -\frac{Q_{in}}{2\pi\epsilon_0} \ln r + c .$$

ϕ_1

ϕ_2

Capacitance / Length

$$= \frac{Q}{\phi_1 - \phi_2} = \frac{2\pi\epsilon_0}{\ln(b/a)}$$