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$$\boxed{\begin{aligned} q' &= -q \cdot \frac{R}{a} \\ b &= \frac{R^2}{a} \end{aligned}}$$

$$\Phi(r,\theta) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{r^2+a^2-2ra\cos\theta}} - \frac{R/a}{\sqrt{r^2+b^2-2rb\cos\theta}}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{r^2+a^2-2ra\cos\theta}} - \frac{R}{\sqrt{r^2a^2+R^4-2R^2ar\cos\theta}}\right]$$

$$Q = 2\pi \int_0^\pi \sigma(R,\theta)\, R^2 \sin\theta \, d\theta.$$

$$= \frac{q}{2} \int_{-1}^{+1} \frac{R(R^2-a^2)}{(R^2+a^2-2aR\mu)^{3/2}} \cdot \frac{d\mu}{(-2aR)}$$

$$= -q \cdot \frac{R}{a}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2(R/a)}{\left| a - \frac{R^2}{a} \right|^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2 R a.}{|a^2 - R^2|^2}$$

$a >> R.$ $\quad F \sim \frac{1}{a^3}$

$a \approx R$ $\quad a = R + \alpha$ $\quad \alpha << R$

$$F \sim \frac{1}{\alpha^2}.$$

$$q' = -q \cdot \frac{R}{a} = -q \cdot \frac{R}{d+R}$$

d

$$\rightarrow -q \quad \text{as } R \mapsto \infty$$

$$b = \frac{R^2}{a} = \frac{R^2}{d+R}.$$

$$b' = R - b = R - \frac{R^2}{d+R} = \frac{Rd}{d+R}.$$

$$= d \quad \text{as } R \mapsto \infty$$

$$\phi(r,\theta,\varphi) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\sqrt{a^2 + r^2 - 2ar\cos\theta}}$$

$$\frac{Rq}{a} \quad \text{at} \quad b = \frac{R^2}{a}$$

$$\phi(r,\theta,\varphi) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Rq/a}{\sqrt{\frac{R^4}{a^2} + r^2 - 2\frac{R^2}{a}\cdot r\cos\theta}}$$

$$\phi\left(\frac{R^2}{r},\theta,\varphi\right) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\sqrt{a^2 + \frac{R^4}{r^2} - 2\frac{R^2}{r}a\cos\theta}}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\left[\frac{\vec{r}+a\hat{k}}{|\vec{r}+a\hat{k}|^3} - \frac{\vec{r}-a\hat{k}}{|\vec{r}-a\hat{k}|^3}\right]$$

$$\frac{1}{|\vec{r}\pm a\hat{k}|^3} = \frac{1}{(r^2+a^2\pm 2az)^{3/2}}$$

$$\simeq \frac{1}{a^3} \qquad a \gg$$

$$\vec{E} = \left(\frac{Q}{2\pi\epsilon_0}\cdot\frac{1}{a^2}\right)\hat{k}$$

$$= E_0\,\hat{k}$$

$$\boxed{E_0 = \frac{Q}{2\pi\epsilon_0 a^2}.}$$

$$\phi_{image} = \frac{1}{4\pi\epsilon_0} \cdot \frac{QR}{a}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{QR}{a}\left(\frac{1}{\sqrt{\frac{R^4}{a^2} + r^2 - 2r \cdot \frac{R^2}{a}\cos\theta}} - \frac{1}{\sqrt{\frac{R^4}{a^2} + r^2 + 2r\frac{R}{a}\cos\theta}}\right)$$

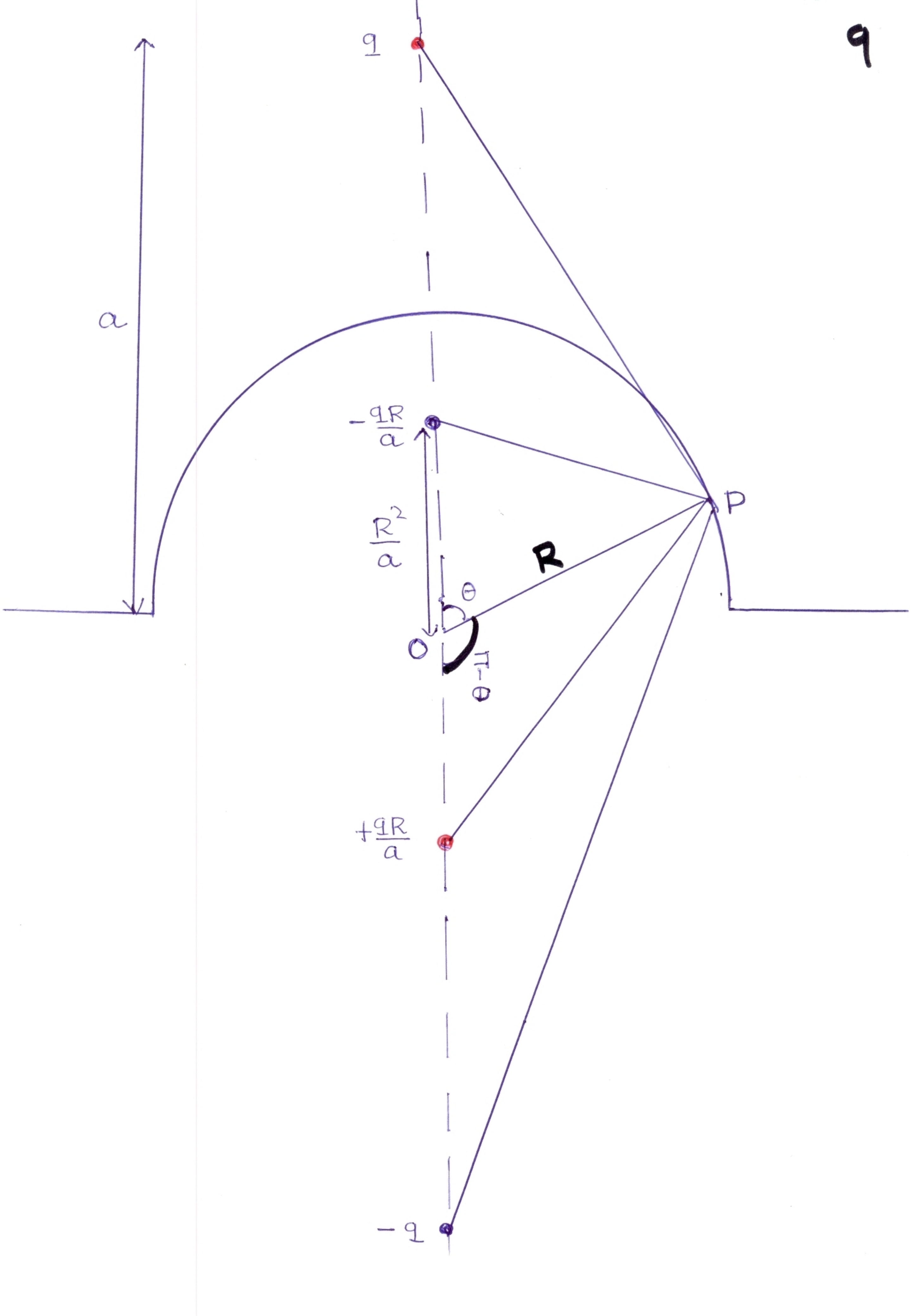
$$= \frac{1}{4\pi\epsilon_0} QR\left[\frac{1}{\sqrt{\underline{\underline{R^4}} + a^2r^2 - 2raR^2\cos\theta}} - \frac{1}{\sqrt{\underline{\underline{R^4}} + a^2r^2 + 2raR^2\cos\theta}}\right].$$

$\underline{a \gg r, R}$

$$= \frac{1}{4\pi\epsilon_0} \frac{QR}{ar}\left[\left(1 + \frac{R^2}{ar}\cos\theta\right) - \left(1 - \frac{R^2}{ar}\cos\theta\right)\right]$$

$$\phi_{images} = E_0 \cdot \frac{R^3}{r^2} \cos\theta$$

$$\boxed{\phi = E_0\left(\frac{R^3}{r^2} - r\right)\cos\theta}$$



q
a
$-\frac{qR}{a}$
P
$\frac{R^2}{a}$
R
θ
O
π−θ
$+\frac{qR}{a}$
−q

$\sigma(r, \theta) =$