Prof. D. Ghoshe
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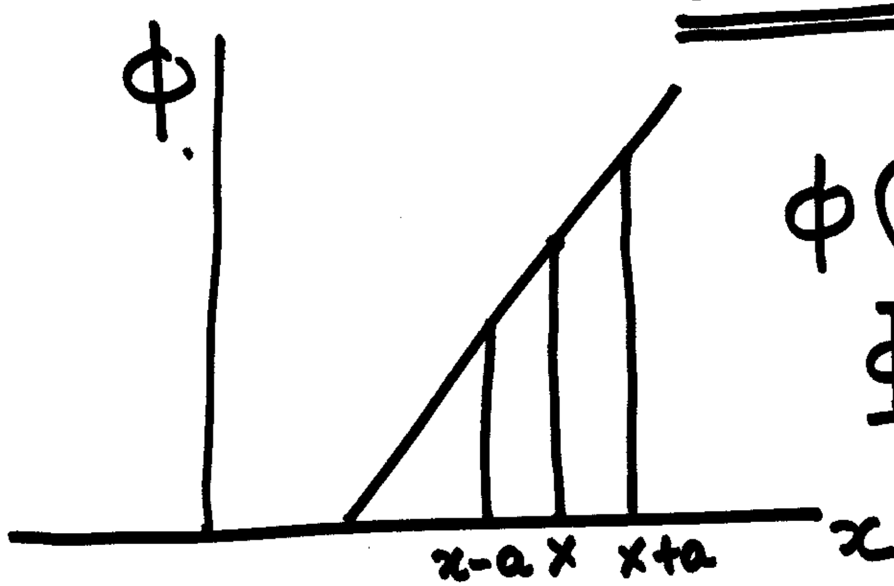
$$\nabla^2 \Phi = 0$$

$$\frac{d^2\phi}{dx^2} = 0 \;; \quad \frac{d\phi}{dx} = m$$

$$\Phi = mx + C$$



$$\phi(x)$$

$$\Phi(x) = \frac{1}{2}\left[\phi(x+a) + \phi(x-a)\right]$$

$$\Phi(x, y) = \frac{a}{4}(x^2 + y^2)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla^2 \phi = a$$

$$\phi(x, y) = \frac{a}{4}(x^2 - y^2)$$

$$\nabla^2 \phi = 0 \Rightarrow \text{Laplace Eqn}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0$$

$$\Phi = X(x)\,Y(y)\,Z(z)$$

$$YZ\frac{\partial^2 X}{\partial x^2}+XZ\frac{\partial^2 Y}{\partial y^2}+XY\frac{\partial^2 Z}{\partial z^2}=0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}=0$$

$$\frac{\partial^2 X}{\partial x^2}=-k_x^2 X \qquad \frac{\partial^2 Z}{\partial z^2}=+k_z^2 Z$$

$$\frac{\partial^2 Y}{\partial y^2}=-k_y^2 Y$$

$$-k_x^2-k_y^2+k_z^2=0$$

$$\frac{\partial^2 X}{\partial x^2} = -k_x^2 X$$

$$X(x) \sim \sin(k_x x)$$

$$X(x = L_1) = 0 \Rightarrow \sin(k_x L_1) = 0$$

$$k_x = \frac{m\pi}{L_1}$$ m is an integer

$$Y(y) \sim \sin(k_y y)$$

$$k_y = \frac{n\pi}{L_2}.$$

$$\frac{\partial^2 Z}{\partial z^2} = +k_z^2 Z \quad : \quad \boxed{Z \sim \sinh(k_z z)}$$

$$k_z = \sqrt{\left(\frac{m\pi}{L_1}\right)^2 + \left(\frac{n\pi}{L_2}\right)^2} \equiv k_{m,n}$$

$$\Phi(x, y, z) = \sum_{m,n=1}^{\infty} C_{mn} \cdot \sin\left(\frac{m\pi}{L_1} x\right) \sin\left(\frac{n\pi}{L_2} y\right) \sinh(k_{mn} z)$$

$$\phi(x, y, L_3) = f(x, y)$$

$$= \sum_{m,n=1}^{\infty} C_{mn} \sin\left(\frac{m\pi}{L_1} x\right) \sin\left(\frac{n\pi}{L_2} y\right) \sinh(k_{mn} \cdot L_3)$$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{1}{P\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P}{\partial\theta}\right)$$

$$+\frac{1}{F\sin^2\theta}\cdot\frac{\partial^2 F}{\partial\phi^2}=0$$

$$\frac{\sin^2\theta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{\sin\theta}{P}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial P}{\partial\theta}\right)$$

$$=-\frac{1}{F}\frac{\partial^2 F}{\partial\phi^2}=m^2.$$

$$\frac{\partial^2 F}{\partial\phi^2}=-m^2F$$

$$e^{\pm im\phi}$$

$$\underline{\underline{e^{im.2\pi}=1}} \Rightarrow \text{m is an } \underline{\text{integer}}$$

$$\nabla^2 \Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Phi}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2}\Phi = 0$$

$$\Phi = R(r)\, P(\theta)\, F(\varphi)$$

$$\frac{PF}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial R}{\partial r}\right) + \frac{RF}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial P}{\partial \theta}\right) + \frac{RP}{r^2 \sin^2\theta} \cdot \frac{\partial^2}{\partial \varphi^2} F = 0$$