Prof D. Ghosh
Lec-10
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No double counting.

$$W = \frac{1}{2}\frac{1}{4\pi\epsilon_0}\sum_{i}\sum_{\substack{j \\ i\neq j}}\frac{q_i}{|\vec{r}_i-\vec{r}_j|}q_j$$

$$= \frac{1}{2}\sum_i q_i \underbrace{\left(\frac{1}{4\pi\epsilon_0}\right)\sum_{j\neq i}\frac{q_j}{|\vec{r}_i-\vec{r}_j|}}$$

$$= \frac{1}{2}\sum_i q_i\,\phi(\vec{r}_i)$$

$$W = \frac{1}{2}\sum_i q_i \varphi(\vec{r}_i)$$

$$\Rightarrow \frac{1}{2}\int d^3r\, \rho(\vec{r})\, \varphi(\vec{r}) \quad \text{——} \; 1$$

$$\operatorname{div}\vec{E} = \frac{\rho}{\epsilon_0}$$

$$= \frac{\epsilon_0}{2}\int d^3r\, (\vec{\nabla}\cdot\vec{E})\, \varphi(\vec{r})$$

$$\vec{E} = -\nabla\phi$$

$$= -\frac{\epsilon_0}{2}\int d^3r\, \underline{\underline{(\nabla^2\phi)\, \phi(\vec{r})}}$$

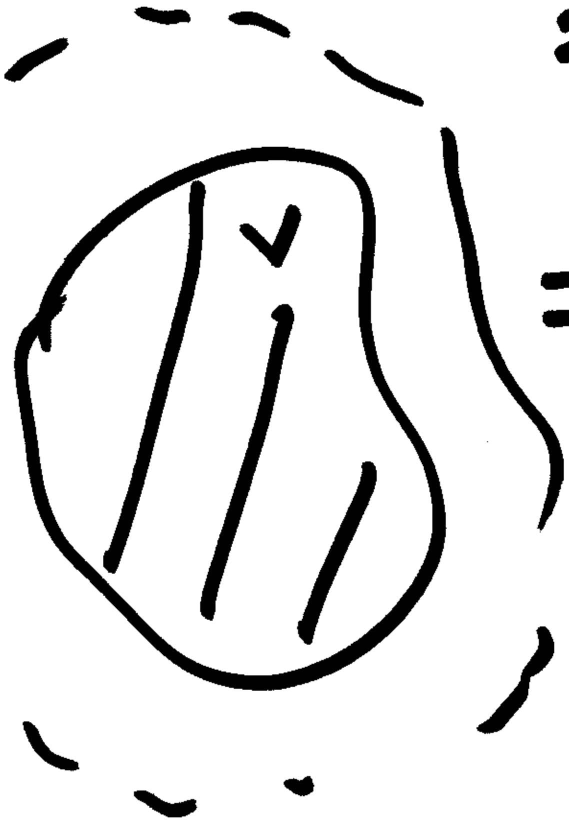
$$\vec{\nabla}\cdot(\varphi\vec{\nabla}\phi)$$

$$= \varphi\nabla^2\phi + \nabla\phi\cdot\nabla\phi$$

$$W = -\frac{\epsilon_0}{2}\int d^3r\,(\nabla^2\phi)\,\phi(\vec{r})$$

$$= +\frac{\epsilon_0}{2}\int_V d^3r\,\Big[\vec{\nabla}\phi\cdot\vec{\nabla}\phi - \underbrace{\nabla\cdot(\phi\nabla\phi)}_{\Downarrow}\Big]$$

$$\underline{\int \phi\,\vec{\nabla}\phi\cdot d\vec{s}} \Downarrow 0$$



$$= \frac{\epsilon_0}{2}\int_{\text{All Space}} |E|^2\, d^3r$$

$$\Downarrow$$

Positive

$$\varphi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \quad (r > R)$$

$$= \underline{\frac{Q}{8\pi\epsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)} \qquad \underline{\text{for} < R}.$$

$$\boxed{\rho = \frac{Q}{\frac{4\pi}{3}R^3}}$$

$$W = \frac{1}{2}\int \rho\,\varphi\, d^3r.$$

$$= \frac{3Q \cdot 4\pi}{8\pi R^3}\int_0^R \frac{Q}{8\pi\epsilon_0 R}\left(3 - \frac{r^2}{R^2}\right) r^2\,dr.$$

$$= \frac{3Q^2}{16\pi\epsilon_0 R^4}\int_0^R \left(3 - \frac{r^2}{R^2}\right) r^2\,dr$$

$$= \frac{3Q^2}{20\pi\epsilon_0 R} \Leftarrow$$

$$W = \frac{\epsilon_0}{2} \int_{\text{All Space}} |E|^2 \, d^3r$$

$$= \frac{\epsilon_0}{2} \left[4\pi \int_0^R \frac{Q^2}{16\pi^2 \epsilon_0^2} \cdot \frac{r^2}{R^6} \, r^2 \, dr + 4\pi \int_R^\infty \frac{Q^2}{16\pi^2 \epsilon_0^2} \cdot \frac{1}{r^4} \cdot r^2 \, dr \right].$$

$$= \frac{3Q^2}{20\pi \epsilon_0 R}$$

$\vec{E}$:

$$\frac{Q}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \quad : r > R$$

$$\frac{Q}{4\pi\epsilon_0} \cdot \frac{\vec{r}}{R^3} \quad : r < R$$

$\underline{r < R}$

$$\varphi(\vec{r}) = \frac{Q}{8\pi\epsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$$

$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\vec{r}}{R^3}. \leftarrow$$

$$W = \frac{\epsilon_0}{2}\int_{\text{Surface}} \varphi(r)\, \vec{E}\cdot\hat{n}\, ds + \frac{\epsilon_0}{2}\int_{\text{Vol.}} |E|^2 d^3r$$

$$= \frac{\epsilon_0}{2} \cdot \varphi(R)\, |\vec{E}(R)| \cdot 4\pi R^2$$

$$+ \frac{\epsilon_0}{2}\int_{\text{Vol.}} \frac{Q^2}{16\pi^2\epsilon_0^2} \cdot \frac{r^2}{R^6}\; 4\pi r^2 dr$$

$$= \frac{3Q^2}{20\pi\epsilon_0 R} \leftarrow$$

$$W = \frac{\epsilon_0}{2}\int |E|^2 \, d^3r .$$

$$\frac{q_1}{4\pi\epsilon_0}\,\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3} + \frac{q_2}{4\pi\epsilon_0}\,\frac{\vec{r}-\vec{r}_2}{|\vec{r}-\vec{r}_2|^3} .$$

$$\left(\vec{E}_1 + \vec{E}_2\right)^2$$

$$\underline{\underline{E_1^2}} \quad , \quad \underline{\underline{E_2^2}} \rightarrow$$

Infinite
Self Energy