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$$\varphi = \frac{q}{4\pi\epsilon_0} \frac{e^{-r/\lambda}}{r}.$$

$$\vec{E} = -\nabla\phi = -\frac{q}{4\pi\epsilon_0}\hat{r}\frac{\partial}{\partial r}\left(\frac{e^{-r/\lambda}}{r}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\hat{r}\frac{1}{r^2}\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}.$$

$$\vec{\nabla}\cdot\vec{E} = \frac{q}{4\pi\epsilon_0}\vec{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\underbrace{\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left[\vec{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\right)\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda} + \frac{\hat{r}}{r^2}\nabla\left[\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}\right]\right].$$

$$\vec{\nabla} \cdot \underline{\underline{\vec{E}}} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot (-\vec{\nabla}\phi) = \frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla^2\phi = -\frac{\rho}{\epsilon_0}}$$

Poisson's Eqn.

$$\nabla^2\phi = 0 \rightarrow$$ Laplace's Eqn.

$$\varphi(\vec{r}) = \frac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}$$

$$\vec{E}(\vec{r}) = -\nabla\phi(\vec{r})$$

$$= -\frac{p}{4\pi\epsilon_0}\nabla\left(\frac{\cos\theta}{r^2}\right)$$

$$= -\frac{p}{4\pi\epsilon_0}\left[-\hat{r}\,\frac{2}{r^3}\cdot\cos\theta + \frac{\hat{\theta}}{r}\left(\frac{-1}{r^2}\right)\cdot\sin\theta\right]$$

$$= \frac{\cancel{p}\,1}{4\pi\epsilon_0}\left[\frac{\hat{r}}{r^3}\;\frac{2\vec{p}\cdot\hat{r}}{r^3} + \frac{p}{r^3}\,\hat{\theta}\,\sin\theta\right]$$

$$= \frac{1}{4\pi\epsilon_0}\;\frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}$$