$$\vec{\nabla} \cdot \vec{V} = \text{div}\,\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Divergence Theorem

$$\boxed{\int \vec{F} \cdot \vec{ds} = \int (\text{div}\,\vec{F})\,dv}$$

$\vec{\nabla} \cdot \vec{F}$ Scalar Field

Net Increase in mass

$$-\left[\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}\right]\underbrace{dx\,dy\,dz}_{d\tau}$$

$$\frac{\partial \rho}{\partial t}\cdot dx\,dy\,dz$$

$$\boxed{\vec{\nabla}\cdot\vec{V}+\frac{\partial \rho}{\partial t}=0}$$

Equation of Continuity

$$\boxed{\vec{\nabla} \cdot \vec{V} + \frac{\partial \rho}{\partial t} = 0}$$

$$\vec{F} = x^2 y \, \hat{i} + x y^2 \, \hat{j} \,.$$

$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow \vec{\nabla} \cdot \vec{V} = 0$$

$$\boxed{\int_S \vec{F} \cdot \vec{ds} = \int_V \operatorname{div} \vec{F} \, dV.}$$

$$\vec{F} = \vec{r}$$

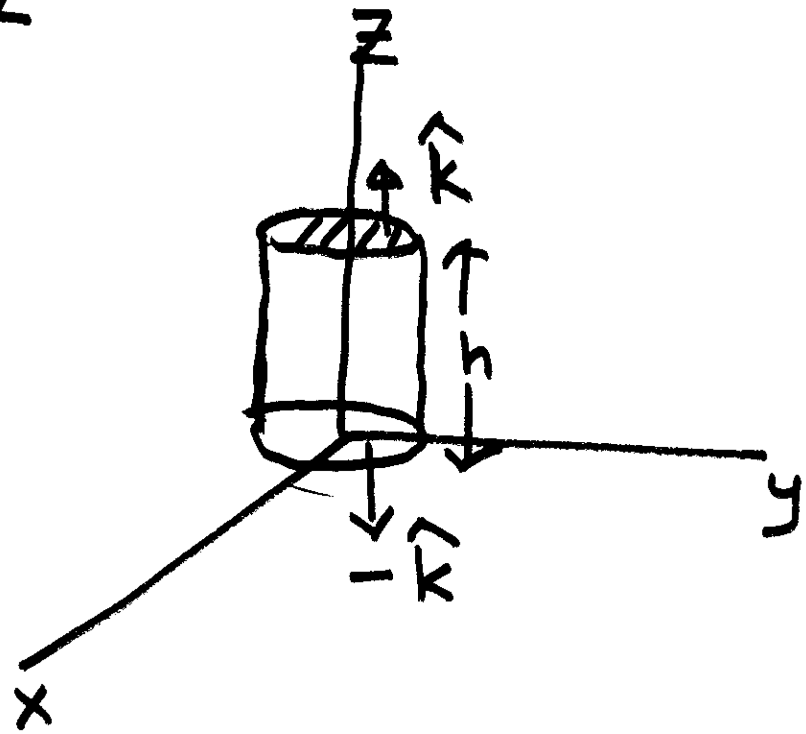
$$\int_{\text{Volume}} \operatorname{div} \vec{r} \, dV = \int_{\text{volume}} 3 \, dV = \underline{\underline{3\pi a^2 h}}$$

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}Z.$$

$$\nabla \cdot \vec{r} = \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z = 1+1+1$$

$$= 3$$

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$



Top: $\displaystyle\int_{Top} \underline{\underline{z}}\, ds$

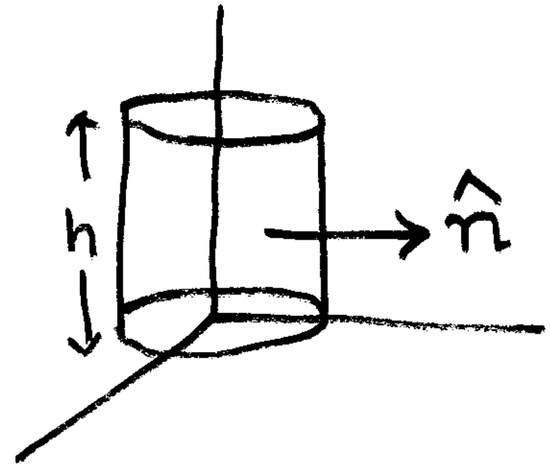
$$= \int_{Top} h\, dS = h \times \pi a^2$$

Bottom $\Rightarrow \displaystyle\int_{Bot} z\, ds$; $z = 0$ for bottom

$= 0$ Top + Bottom $\Rightarrow \underline{\underline{\pi a^2 h}}$

5

$$\hat{n} = \frac{\hat{\imath}\,x + \hat{\jmath}\,y}{\sqrt{x^2+y^2}} .$$



$$= \frac{\hat{\imath}\,x + \hat{\jmath}\,y}{a}$$

$$\vec{F}.\hat{n} = (\hat{\imath}\,x + \hat{\jmath}\,y + \hat{k}\,z)\cdot\left(\frac{\hat{\imath}\,x + \hat{\jmath}\,y}{a}\right)$$

$$= \frac{x^2+y^2}{a} = \frac{a^2}{a} = a .$$

$$\int \underline{\underline{\vec{r}.\hat{n}}}\, ds = a\int ds$$

$$= a \times 2\pi a h$$

$$= 2\pi a^2 h$$

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$= 2 + 2y + x$$

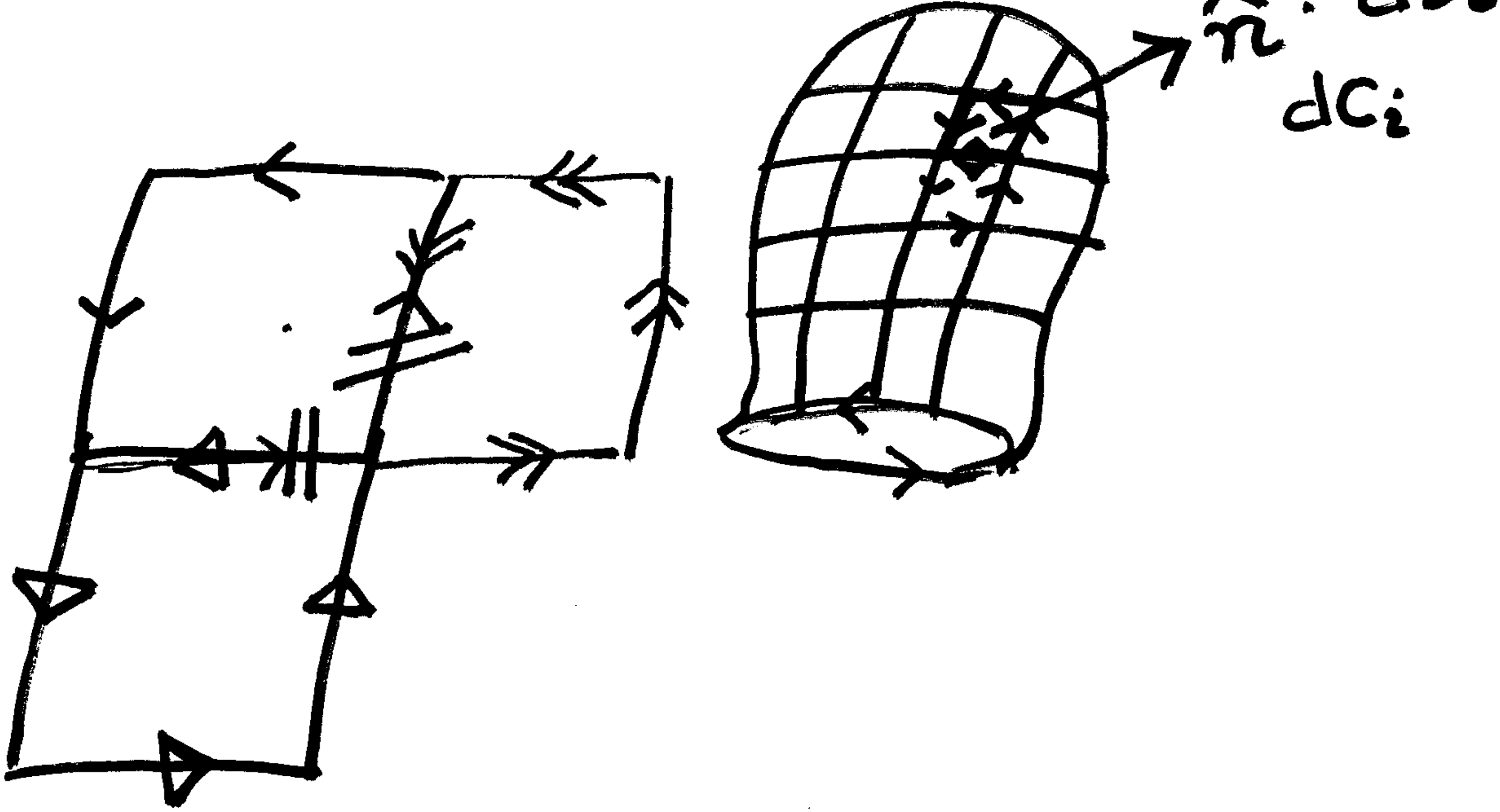
$$\iiint dx\, dy\, dz\, (2 + 2y + x).$$

$$= \int_0^1\!\!\int_0^1 dx\, dy\, (\underline{\underline{2}} + 2y + \underline{\underline{x}})$$

$$= 2 + 2 \times 1 \times \frac{1}{2} + 1 \times \frac{1}{2}$$

$$= 2 + 1 + \frac{1}{2} = \frac{7}{2}$$

CURL



$$\oint_C \vec{F}\cdot dl = \sum_i \left(\frac{\oint_{C_i} \vec{F}\cdot dl}{\Delta S_i} \right) \Delta S_i$$

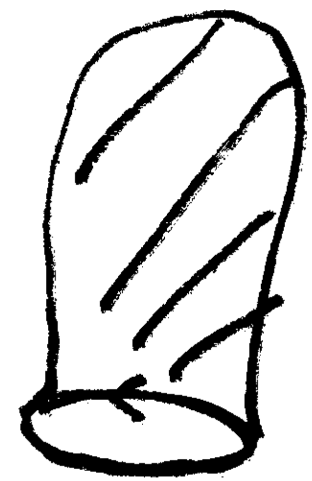
$$\text{Curl}\,\vec{F} \equiv \frac{\oint_{C_i} \vec{F}\cdot \vec{dl}}{\Delta S_i}\,\hat{n}_i$$

In the Limit

$\Delta S_i \to 0$

$$\int_C \vec{F}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{F})\cdot\vec{ds}$$

OPEN SURFACE



Stoke's Theorem