Prof. D. Ghosh
Lec-1- Lab-21-7-10

$3x^2y$ $\quad \vec{c} = (-2,1)$

$\vec{ds} = 3\hat{i} + 4\hat{j}$ $\quad \hat{u} = \frac{3\hat{i}+4\hat{j}}{5}$

$$D_u f(\vec{c}) = \lim_{h\to 0} \frac{f(\vec{c}+h\vec{u}) - f(\vec{c})}{h}$$

$$= \lim_{h\to 0} \frac{f\left(-2+h\frac{3}{5}, 1+h\frac{4}{5}\right) - f(-2,1)}{h}$$

$$\varphi = c \qquad t$$

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

$$\varphi(x(t), y(t), z(t)) = c$$

$$\vec{r}'(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

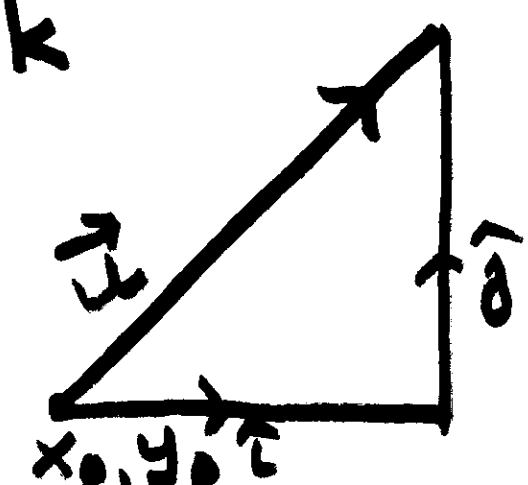
$$\frac{d\varphi}{dt} = 0 = \frac{\partial\varphi}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial\varphi}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial\varphi}{\partial z}\frac{dz}{dt}\cdot$$

$$= \nabla\phi \cdot \vec{r}' = 0$$

$$\nabla\Phi = \text{grad}\,\Phi$$

$$= \hat{i}\frac{\partial\Phi}{\partial x} + \hat{j}\frac{\partial\Phi}{\partial y} + \hat{k}\frac{\partial\Phi}{\partial z}.$$

$$\hat{u} = a\hat{i} + b\hat{j} + c\hat{k}$$

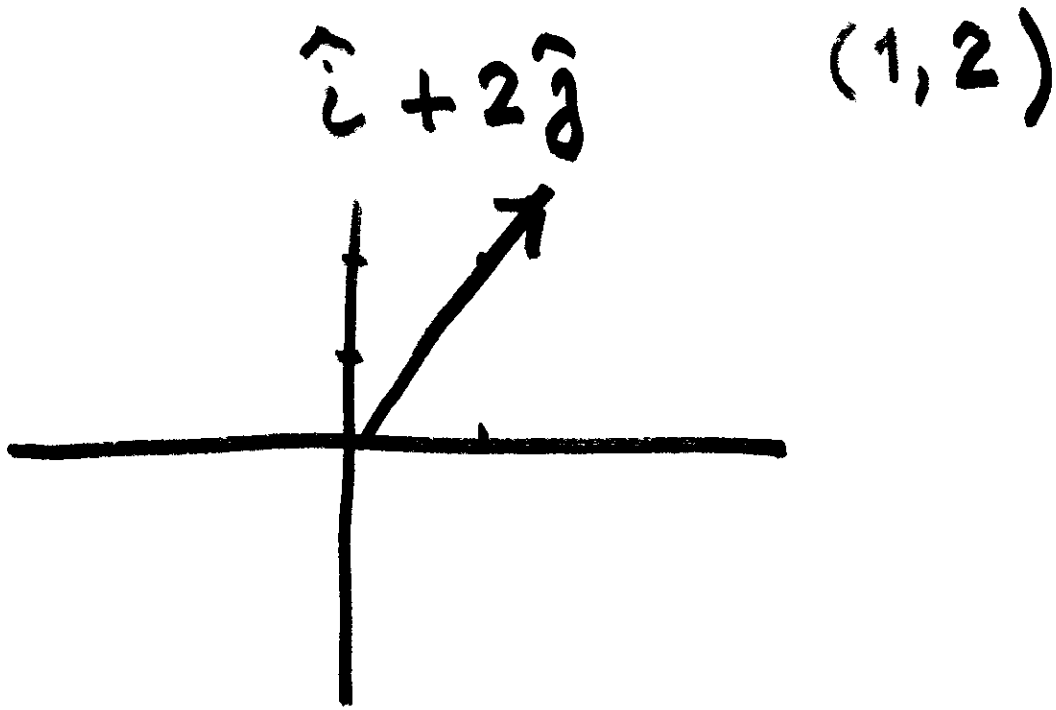


$$\frac{d\Phi}{ds} = \frac{\partial\Phi}{\partial x}\frac{dx}{ds} + \frac{\partial\Phi}{\partial y}\frac{dy}{ds} + \frac{\partial\Phi}{\partial z}\cdot\frac{dz}{ds}$$

$$= \frac{\partial\Phi}{\partial x}a + \frac{\partial\Phi}{\partial y}b + \frac{\partial\Phi}{\partial z}c = \vec{\nabla\Phi}\cdot\hat{u}$$

$\hat{\imath} + \alpha\hat{\jmath}$ $\qquad y = \alpha x$

$$\frac{d\varphi}{ds} = \frac{1+2\alpha}{\sqrt{1+\alpha^2}} \qquad \alpha = 2$$



~~$\phi(x^2+y$~~

$$\boxed{\phi(x,y) = x^2+y^2} \quad (1,2)$$

(i) $\hat{i}+2\hat{j}$ $\qquad y = 2x; \quad \frac{dy}{dx} = 2$

$$\frac{d\phi}{ds} = \frac{\partial\phi}{\partial x}\cdot\frac{dx}{ds} + \frac{\partial\phi}{\partial y}\cdot\frac{dy}{ds}$$

$$= 2x\frac{dx}{ds} + 2y\frac{dy}{ds}$$

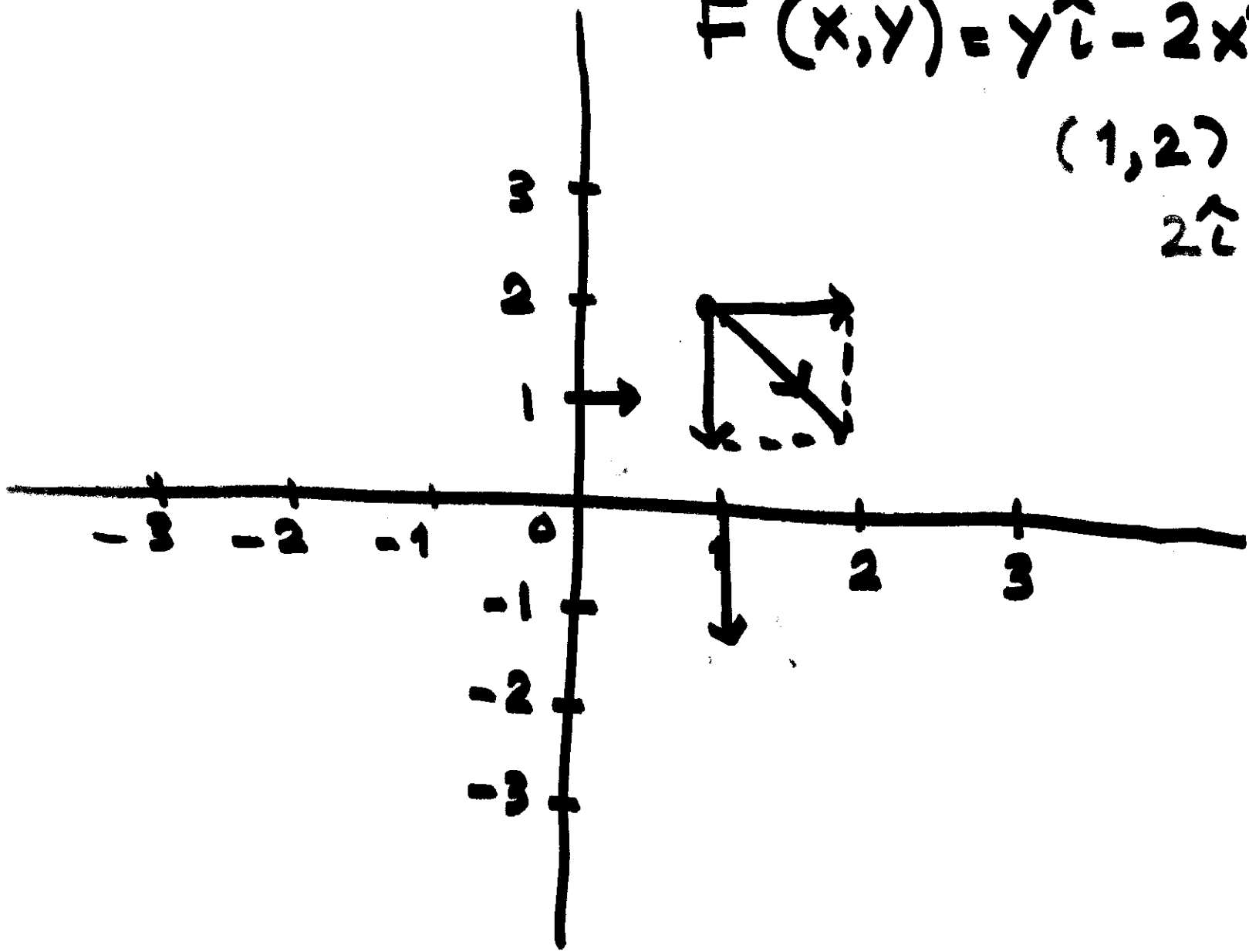
$$ds = \sqrt{dx^2+dy^2}$$

$$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+4} = \sqrt{5}$$

$\vec{F}(x,y) = y\hat{i} - 2x\hat{j}$

$(1,2)$

$2\hat{i} - 2\hat{j}$



$$f(x)$$

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$