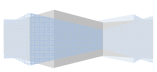
Module 9

Deformation of Pure Metals II

Lecture 9

Deformation of Pure Metals II



Keywords: Mechanisms of plastic deformation, Slip, Twin, Stress-strain diagram of single crystal, Crystal rotation during plastic deformation, Strength of an ideal crystal, Deformation of polycrystalline material, Preferred orientation or texture

Introduction

In the last module we learnt about elastic deformation of metals and the relation between the stress which is the cause and the strain which is the effect or the outcome. Elastic deformation is temporary. It disappears when the stress is withdrawn, as against this plastic deformation is permanent. It occurs only when the stress exceeds a characteristic value called yield strength. This is a material property. In this lecture we shall talk about the characteristics of plastic deformation; its mechanisms and its effects on the structure & properties of metal. Pure metal is ductile and malleable. It can be rolled into thin foils or drawn into fine wires. Aluminum foils for packaging and fine copper wires for electrical conductors could be made primarily because of this unique feature of metals. Metals are made of several grains (crystals) that are oriented at random. In spite of the difference in the orientation between two neighboring grains there exists a strong bond between these. When it undergoes plastic deformation the shape of the grain and the boundary change but the continuity across the grain is maintained. There is little change in its volume or crystal structure. This is one of the major differences between elastic and plastic deformation. Let us begin this module with the mechanisms of plastic deformation of a single crystal.

Deformation of single crystal:

There are two ways a crystal can deform without any change in its crystal structure. These are slip and twin. Figure 1 illustrates how deformation takes place on application of stress by slip. If the stress is less than the yield strength of the metal the atoms are just pulled apart. The lattice parameter increases along the direction of stress. When it exceeds the yield strength a part of the crystal slips over the other. This takes place on a plane on which the shear stress is the maximum.

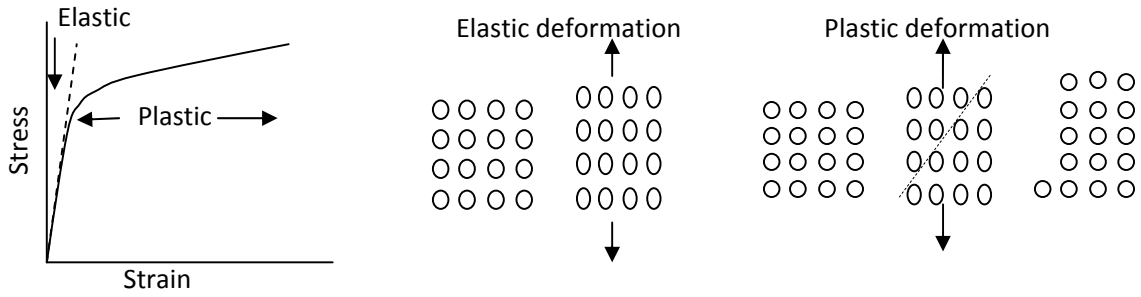


Fig 1: Illustrates the difference between elastic & plastic deformation. Atoms move apart along the direction of stress but come closer along the direction perpendicular to the applied stress during elastic deformation. When the stress is withdrawn the atoms come back to their previous positions. Whereas during plastic deformation the atoms on either sides of a plane on which the shear stress is the maximum slide over one another. Displacement occurs in multiples of atomic distance on the slip plane

along the slip direction. The atoms do not come back to their initial positions when the stress is withdrawn. The stress strain plot starts deviating from linearity when plastic deformation sets in.

Plastic deformation by slip can occur only on certain planes and along specific directions on the plane. The combinations of slip planes and directions on which slip can take place are known as slip system. It depends on the crystal structure. Usually the close packed planes are the slip planes since they happen to be the most widely spaced planes. Slip directions are the close packed directions. Table 1 gives the indices of slip planes and directions for the three most common crystal structures of metals.

Table 1: Lists of slip system for 3 most common crystal structures

Crystal lattice	Slip plane	Slip Direction	No. of slip system
FCC	{111}	<110>	12
HCP	{0001}	$< 2\bar{1}10 >$	3
BCC	{110} {112} {123}	<111>	48

It is worth noting that slip can occur only due to stress acting on the slip plane along the slip direction. Even if the applied stress is tensile along a given direction one has to find its component along the slip direction on the slip plane to know if it can induce plastic deformation. Such stresses are known as resolved shear stress. Slip occurs when the magnitude of this resolved shear stress exceeds a critical value. This is known as the critical resolved shear stress. Slip can also be visualized as a simple shear.

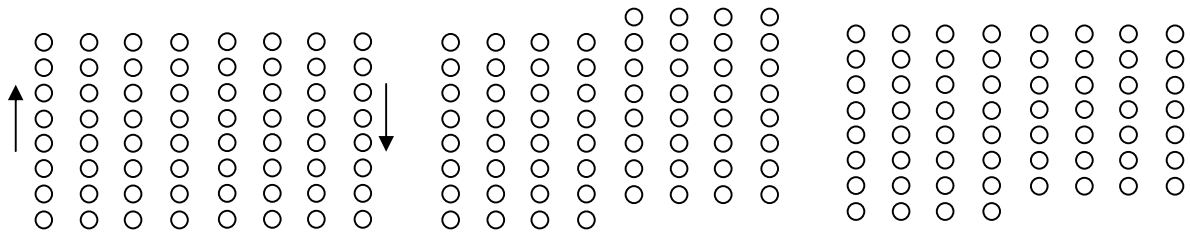


Fig 2: Illustrates atomic arrangement before and after deformation by slip (simple shear). The process would leave a slip trace (step) on the surface. Atoms on both sides of the slip plane are arranged in identical fashion. If the top surface is polished the sign of deformation is totally removed. The best way to see the signs of deformation due to slip is to take a polished sample, deform and look under microscope without tampering (or polishing the surface) the surface.

The bulk of plastic deformation takes place by slip. The main features of deformation by this mechanism are (i) no change in crystal structures (ii) no change in volume or lattice parameter (iii) no change in crystal orientation (iii) all the atoms above the slip plane move by identical distance on the slip plane along the slip direction (iv) the net displacement is in multiples of atomic spacing.



Fig 3 Sketches showing the effect of stress on a polished polycrystalline sample under stress. When the stress exceeds the yield stress the sample under goes permanent deformation. Note the change in the shapes of the grains and the sample. There is no change in volume. Deformation leaves marks on the grains as shown. These are the traces of the slip planes (lines of intersection of the slip plane and the top face of the sample). If the top face is polished the traces would vanish.

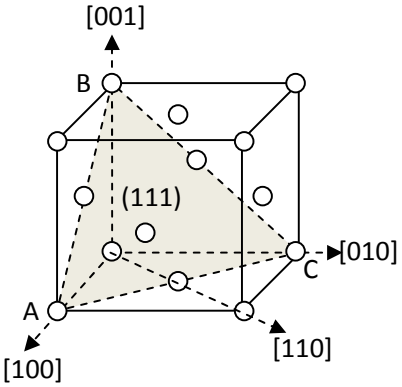


Fig 4: Sketch showing the location of atoms in an FCC unit cell. Look at the close packed plane (111) and the indices of the lines joining atoms at face centers with those at the corners. All of these are close packed directions. There are 3 close packed directions in one slip plane. Since there are 4 close packed planes in FCC crystal the total number of slip planes = 4 x 3 =12.

Problem 1: Figure 4 shows one (111) plane. What are the indices of the directions AB, BC & CD?

Answer: $[\bar{1}01]$, $[01\bar{1}]$ & $[1\bar{1}0]$. Note that if (hkl) denotes plane and [uvw] denotes direction lying on the plane then hu+kv+lw = 0. This is true for orthogonal axes.

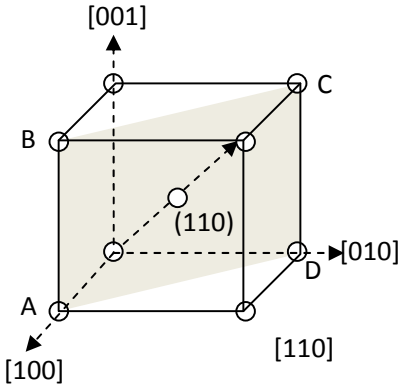


Fig 5: Sketch showing the location of atoms in a BCC unit cell. Look at the close packed plane (110) and the indices of the line joining atoms at the body centre with those at the corners [111]. All of these are close packed directions. There are 2 close packed directions in one slip plane. Since there are 6 close packed planes in BCC crystal the total number of slip planes = 2 x 6 =12.

Problem 2: What are the indices of the directions AC & BD?

Answer: $[\bar{1}11]$ & $[\bar{1}1\bar{1}]$. Here too the condition hu + kv + lw = 0 is satisfied.

Note that FCC is a close packed structure. Density of atoms / unit area is much more in (111) plane than any other planes. (111) planes are also planes having maximum inter planar distance. Therefore these happen to be the only slip plane. The packing density of BCC crystal is not so large. Density of atoms in

(110) planes is only a little more than (112) and (123) planes however [111] direction in BCC lattice is as close packed as [110] in FCC. Therefore although the slip direction in BCC happens to be [111] slip may also occur on (112) & (123) planes. The number of slip system in BCC crystal is thus much more than those in FCC crystal. This is also the reason why BCC metals like iron shows wavy slip lines.

Problem 3: Show that the number of slip system in BCC metal is 48.

Answer: Note that on each (112) there is only one close packed direction [111]. There are 12 planes having similar indices (packing density). Thus the number of slip system of type {112} <111> is 12. Number of planes of type {123} is 24 and the number of <111> direction lying on it is one. Thus the number of slip system of type {123} <111> is 24. Add all the three to get 48.

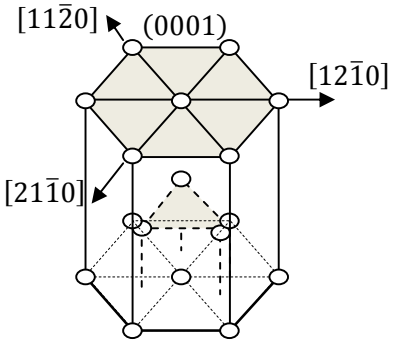


Fig 6: Sketch showing atomic arrangement in an HCP crystal. Basal plane (0001) is the slip plane. There is only one basal plane. The slip direction is $[21\bar{1}0]$. There are 3 such directions in one basal plane. There are only 3 slip systems. Out of these only two are independent.

Numbers of independent slip systems:

Changes in shape or deformation can only take place by shear or slip (both are same). A polycrystalline metal is known to have excellent ability to deform into any desired shape. This is possible if all the crystals of which this is made can undergo any arbitrary deformation. How many slip systems a crystal must possess to satisfy this? To answer this we need to know about various possible components of strains. The total strain (ε_{ij}) is made of elastic (ε_{ij}^{e}) and plastic (ε_{ij}^{p}) components.

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \qquad (1)$$

Let us consider the plastic strain only since we are looking at large deformation. The magnitude of elastic strain is always negligibly small in comparison to that of plastic strain. Strain is a second rank symmetric tensor. It has 6 components as shown by equation (2). .

$$\begin{bmatrix} \varepsilon_{11}^{p} & \varepsilon_{12}^{p} & \varepsilon_{13}^{p} \\ \varepsilon_{12}^{p} & \varepsilon_{22}^{p} & \varepsilon_{23}^{p} \\ \varepsilon_{13}^{p} & \varepsilon_{23}^{p} & \varepsilon_{33}^{p} \end{bmatrix} \qquad (2)$$

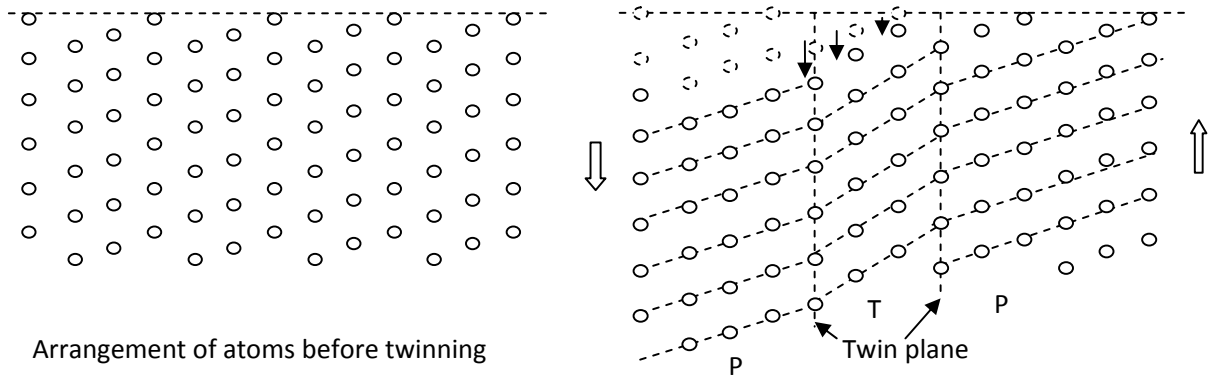
The sum of the diagonal element represents total volume strain. This is an invariant. It means that the volume strain does not depend on the choice of reference axes used to represent the strain at a point. However there is no change in volume during plastic deformation. This suggests that:

$$\varepsilon_{11}^{e} + \varepsilon_{22}^{p} + \varepsilon_{33}^{p} = 0 \qquad (4)$$

The number of independent strain components is therefore 5. Each of these can occur due to slip on a slip specific system. This is why 5 independent slip systems are necessary for any arbitrary deformation. Since both FCC & BCC metals have several slip systems to choose from they have excellent ductility. Gold, silver & aluminum can be rolled down to extremely thin foils. All of them have FCC structure.

Problem arises in the case of metals having HCP structure (Zn, Mg, Zr, Ti). Slip takes place only on the basal plane along any of the three close packed directions. It may be noted that vector addition of the two slip directions gives the third. Thus only two of the 3 slip systems can be considered to be independent. In fact ideal HCP structures have relatively poor ductility. In case of deviations from ideal HCP structure slip is known to occur on prism & pyramid planes. However the slip direction is still the same. This provides additional choice of slip systems from which selection of 5 independent slip systems becomes possible.

Apart from slip there is an altogether different mechanism of plastic deformation. This is called twinning. This too occurs on a specific crystal plane in such a fashion that the deformed part becomes a mirror image of the parent crystal. Unlike slip the movement or atomic displacement is a function of its distance from the twinning plane. Like slip this too occurs due to shear stress.



Arrangement of atoms before twinning

Arrangement of atoms after twinning

Fig 7 Sketch illustrating how the atomic arrangement changes on either sides of a twin plane after deformation. Left hand figure shows the positions of atoms before twinning. The right hand figure shows the positions of atoms after twinning. This also shows previous locations of atoms by dotted circles. These are the locations where there are no atoms now. Note the arrangement of atoms on either side of the twin planes. It exhibits reflection symmetry. Dotted lines indicate one of the close packed directions. These are differently oriented within the twin. This shows that twinning is accompanied by a change in crystal orientation. Therefore on polishing the sign of twin still remains.

Table 2 gives a list of twin plane and direction for the 3 most common crystal structures of metals. The twin plane and direction remain undistorted. However there is a change in the orientations of the other planes and directions within the twin.

Table 2: List of twin planes & directions

Crystal	Twin plane	Twin direction
FCC	{111}	<112>
BCC	{112}	<111>
HCP	$\{10\bar{1}2\}$	$< 10\bar{1}1 >$

Deformation during twinning takes place only within the twinned part of the crystal. However the magnitude of deformation is large. For example the magnitude of displacement in FCC crystal is $\frac{a}{6} < 112 >$ over a distance of $\frac{a}{3} < 111 >$. Therefore the magnitude of shear strain is given by:

$$\gamma = \frac{\frac{a}{6}\sqrt{1+1+4}}{\frac{a}{3}\sqrt{1+1+1}} = \sqrt{\frac{3}{6}} = 0.707 \qquad (5)$$

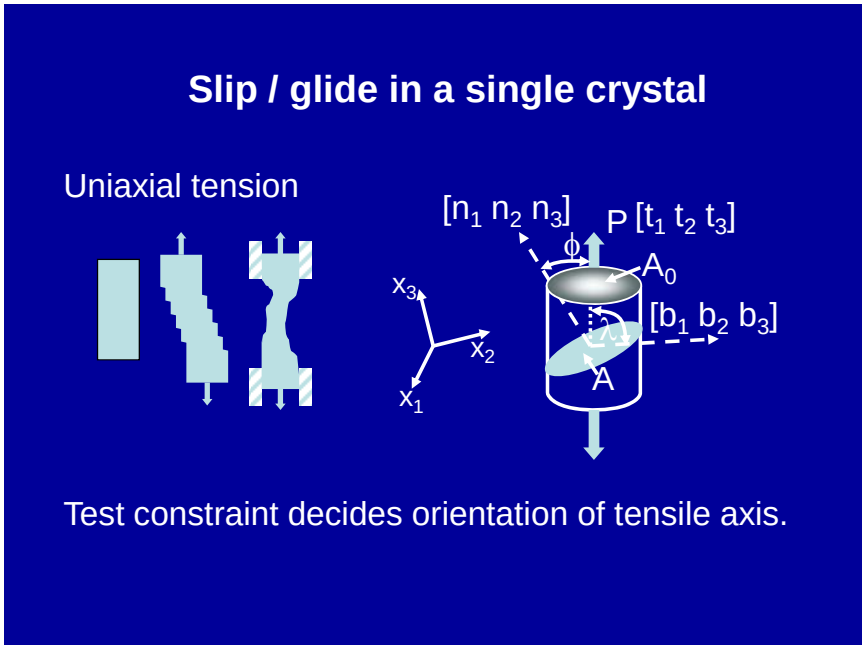
Unlike slip twinning is noisy and fast. One of the best examples of such a feature is that of tin cry (It has BCT structure). You can hear the noise it makes tin makes during deformation. The stress strain plot also shows serration. It is not smooth. HCP crystal has limited slip system. It needs additional deformation modes. The orientation of the deformed region is different from the matrix. As a result it may now be favorably oriented to undergo slip. Therefore twinning is very common in HCP crystal. Twinning (deformation) also takes place in metals having BCC structure. However it is not so common during plastic deformation of FCC crystal. Table 3 gives a comparison between the two modes of plastic deformation.

Table 3: A comparison of slip & twin the two modes of plastic deformation

Features	Slip	Twin
Deformation rate	Slow (time ~ milliseconds)	Fast (time ~ microseconds)
Stress – strain plot	Smooth	Shows serrations
Acoustic emission (sound)	Quiet	Noisy
Displacement of atoms	Same for all planes above the slip plane	It is proportional to its distance from the twin plane
Orientation relation	Identical on both sides of the slip plane	Twinned part is a mirror image of the parent crystal
Effect of polishing on the signs of deformation on the surface	Disappears	Does not disappear

Single crystal deformation by slip (glide): The slide 1 illustrates how slip takes place in a crystal when it is pulled in tension. The left hand side of this illustration shows that if the crystal glides on a plane the

tensile axis is likely to shift. However the the grips of the testing machine will not allow it to happen. The crystal would therefore rotate. The indices of the tensile axis would change. How does this happen? Does it depend on the orientation of the crystal? We shall learn about it now.



Slide 1

Critical resolve shear stress (CRSS):

The sketch on the right hand side of slide 1 illustrates how the tensile axis P [t_1 t_2 t_3] is oriented with respect to the slip plane A whose normal [n_1 n_2 n_3] subtends an angle ⍰⍰with P and ⍰ is the angle between P and the slip direction [b_1 b_2 b_3]. Note the orientation of the orthogonal reference axes x_1 x_2 x_3. The direction x3 points along {n_1 n_2 n_3] and x2 is along [b_1 b_2 b_3]. A_0 denotes the cross section area of the crystal. The driving force for glide (slip) is the resolved stress acting along the slip direction on the slip plane. The expression for this can be derived as follows:

$$RSS = \tau = \frac{Pcos\lambda}{A} = \frac{Pcos\lambda}{A_0/cos\phi} = \left(\frac{P}{A_0}cos\lambda cos\phi\right) = \sigma cos\lambda cos\phi \qquad (6)$$

Note that ⍰ is the applied tensile stress and A is the cross section area of the slip plane. Slip or permanent deformation takes place when ⍰ exceeds a critical value ⍰$_c$. This is known as Critical Resolved Shear Stress. The equation 6 provides a relation between tensile and shear yield stress of a crystal. Note that CRSS is a material property of the crystal. This does not depend on the size geometry and orientation of the crystal. However the tensile yield strength of the crystal depends on its orientation. This is given by:

$$\sigma_y = \tau_c/(cos\lambda cos\phi) \qquad (7)$$

The term (cos⍰cos⍰⍰ represents the orientation relationship of the slip system with respect to the tensile axis. It is commonly known as Schimd's factor. It can have a maximum value of 0.5. Figure 8 shows a sketch of resolved stress versus resolved shear strain. It has several distinct stages. Glide occurs when ⍰⍰> ⍰$_c$. Initially when slip takes place on a single system there is little work hardening. This is known as the period of easy glide. With deformation the tensile axis would rotate. The orientation factor would change so does the resolved shear stress. When the orientation factor on another slip system becomes equally favorable glide occurs simultaneously on the two slip system. This stage is known as duplex slip. As a result strain hardening rate becomes much higher. At a later stage when as result of changing orientation factor several other slip planes become operative there is strain softening (the slope of the stress strain plot decreases). This stage is known as cross slip.

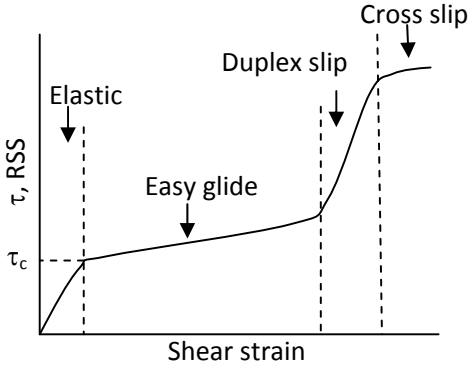
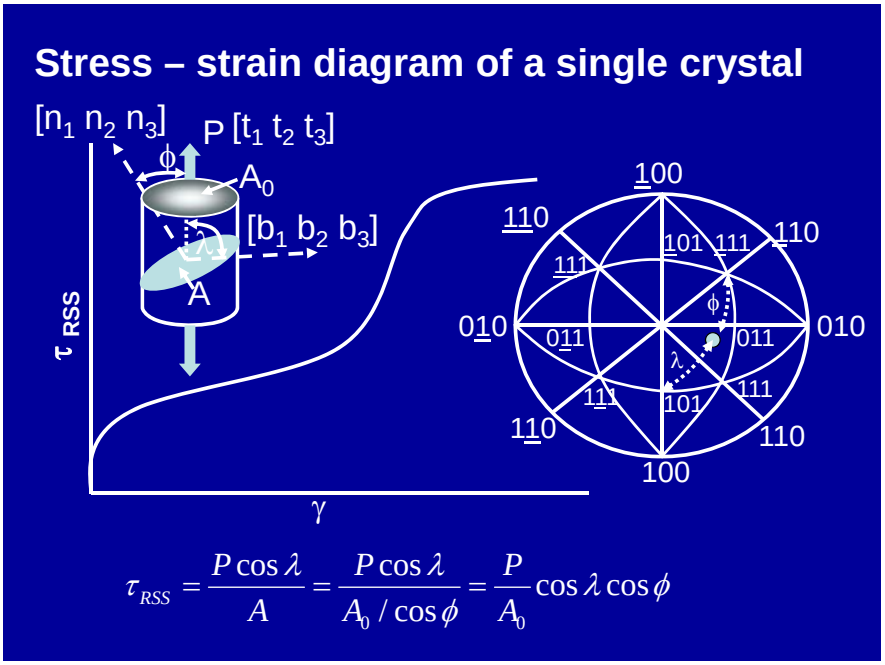


Fig 8: Sketch showing different stages of deformation of a single crystal. Barring elastic stage where the movement of atoms is less than one atomic distance the rest are permanent. The shape of the curve is a function of the orientation of the crystal.



Slide 2: Note the location of tensile axis on a standard projection.

Slide 2 shows the nature of stress strain plot of a single crystal along with 001 standard projection where the location of the tensile axis is shown with respect to a specific slip system. In this case underline

denotes bar. For example $\bar{1}\bar{1}0$ has been represented as 11̲0 on the standard projection. The projection includes the locations of the poles of {111} planes and the slip directions <110>. They constitute the slip system of FCC crystal. There are 12 slip systems. The standard projection in slide 2 indicates the angles □ & □□with respect to slip plane normal $(\bar{1}11)$ & slip direction [101]. The slip system for which the Schmid's factor is the highest is the most favorable slip system. While comparing its values consider its magnitude only (ignore its sign). The sign would help identify the direction in which it glide is likely to take place. For example if Schmid's factor is positive when slip occurs along [101] whereas it would slip along $[\bar{1}0\bar{1}]$ if it is negative. The expression for the estimation of Schmid's factor on the basis of the notations used to denote tensile axis, slip plane and slip direction can be written as follows:

$$SF = cos\phi cos\lambda = \frac{(n_1t_1+n_2t_2+n_3t_3)}{\sqrt{n_1^2+n_2^2+n_3^2}} \frac{(b_1t_1+b_2t_2+b_3t_3)}{\sqrt{b_1^2+b_2^2+b_3^2}\sqrt{(n_1^2+n_2^2+n_3^2)}} \qquad (8)$$

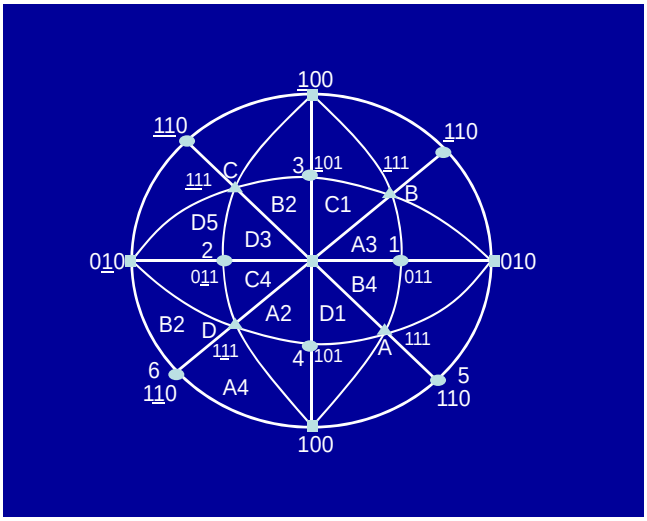
Estimation of Schmid factor for a given tensile axis for different slip system

t1	t2	t3							
1	2	3							
h	k	l		u	v	w	cos f	cos l	m
1	1	1		-1	1	0	0.926	0.189	0.175
1	1	1		-1	0	1	0.926	0.378	0.35
1	1	1		0	-1	1	0.926	0.189	0.175
-1	1	1		1	1	0	0.617	0.567	0.35
-1	1	1		1	0	1	0.617	0.756	0.467
-1	1	1		0	-1	1	0.617	0.189	0.117
-1	-1	1		0	1	1	0	0.945	0
-1	-1	1		1	0	1	0	0.756	0
-1	-1	1		-1	1	0	0	0.189	0
1	-1	1		1	1	0	0.309	0.567	0.175
1	-1	1		-1	0	1	0.309	0.378	0.117
1	-1	1		0	-1	1	0.309	0.189	0.058

Slide 3: A table showing the magnitudes of SF (denoted by m) for FCC crystal when the tensile axis is [123]. (hkl) are the indices of slip plane whereas [uvw] are the indices of slip direction. Note that the magnitude of m is the highest for the slip system $(\bar{1}11)\{101\}$. The equation 8 has been used to find SF.

The above procedure to find the slip system for a cubic close packed crystal loaded along a direction [UVW] may look a bit complicated. It involves estimation of Schmids factor (SF) for a large number of slip systems. Some of these may be negative as well. The most favorably oriented slip system has the highest absolute value for SF (ignore the sign). There is a simple rule that may help one to find this quickly. It is known as OILS rule. The mnemonics is derived from the phrase 'zer**O** **I**ntermediate **L**owest **S**ign'. The first two letters help decide the location of 0 within the indices of the slip direction <110> in case of FCC crystal (slip plane in the case of BCC). Let us use this to find the indices for the slip direction if the tensile axis is oriented along [213]. The intermediate Miller index for this direction is 2. Therefore the slip direction is [011]. **L**owest **S**ign helps identify which index for the slip plane (slip direction in the case of BCC crystal) should have a negative sign. In this case the lowest index of the tensile axis is

1.Therefore the slip plane is$(1\bar{1}1)$. You may use this procedure to show that the most favorable slip system if [123] is the tensile axis is $(\bar{1}11)\{101\}$.



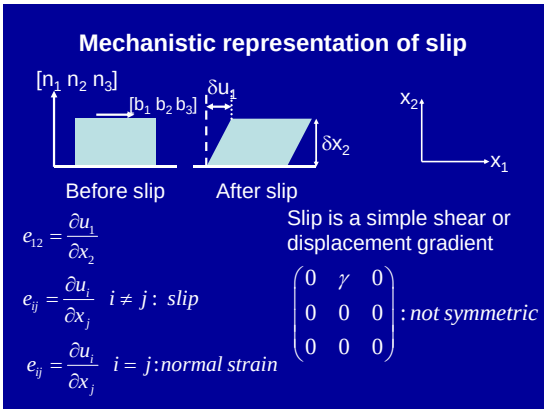
Slide 4: A 001 standard projection showing the locations of the poles of slip planes {111} and slip directions <110>. Symbols A, B, C & D have been used to denote four slip planes and numbers 1-6 have been used for 6 slip directions. Consider the locations within the stereographic triangle joining the poles 001, 111 & 011. Slip system having the highest SF for any crystal whose tensile axis lies within this is B4.

The standard projection shown in slide 4 indicates the choice of slip systems depending on the location of the tensile axis within various stereographic triangles. Only a few of these are shown. You may try to find out the slip system for the rest of the stereographic triangles.

What is the slip system for tensile axes (poles); lying along one of the sides of the stereographic triangles? Consider a pole lying on the line joining 001 with 011. In this case SF for two slip systems A3 & B4 are identical. Therefore slip is likely to occur on both of these. This is known as duplex slip. If 001 is the tensile axis then there are eight slip systems having identical SF. All of them are favorably oriented for slip.

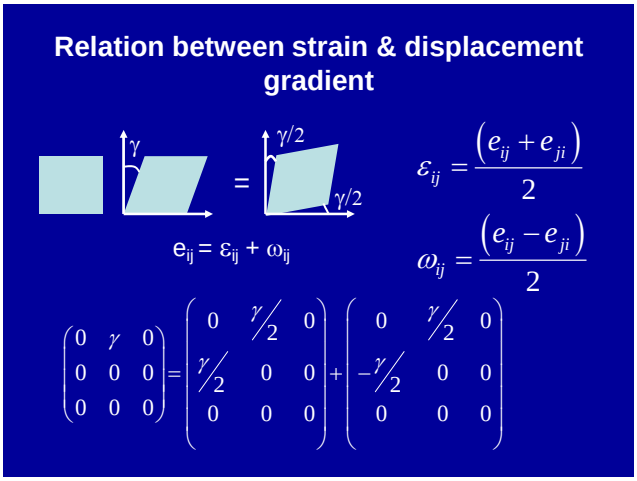
Mechanistic representation of slip:

Slip is a process of simple shear. This results in a change in the shape of the object without any change in its volume. Let the cross section of a crystal be a rectangle before slip. After slip it becomes a parallelogram. This is illustrated in slide 5.



Slide 5: Note that the magnitude of shear displacement along the slip direction (b) on plane (n) is δu_1 over a distance δx_2. Thus the shear strain is given by $\gamma = \frac{\delta u_1}{\delta x_2}$. This may be taken to be the displacement gradient. It represented as e_{ij}. For shear components the subscripts are different ($i \neq j$). Displacement gradient matrix has only one nonzero term when represented with respect to an axes ox as shown. OX_1 points along b the slip vector & OX_2 points along plane normal n.

Simple shear represented by a non-symmetric matrix in slide 5 can be visualized to be made of two parts of which one is symmetric strain matrix and the other an anti-symmetric matrix representing rotation. This is illustrated in slide 6.

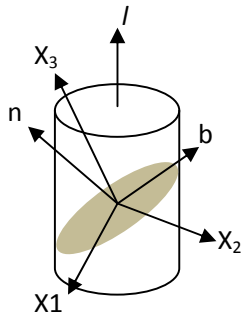


Slide 6: Shows the relation between displacement gradient matrix and strain matrix. The sketch shows the shape of the body before slip, after slip and another after it has undergone a rotation about an axis perpendicular to the plane. Note that the plane of the slide contains both slip vector and slip plane normal. The matrix denoting simple shear has been broken into two matrices. The first represents pure shear. The magnitude of shear strain is $\gamma/2$. The second matrix denotes rotation. The magnitude of rotation is $\gamma/2$.

Therefore during tensile test on a single crystal the indices of the tensile axis may change due to slip. The introduction of matrix notations helps one find out the indices of the new axis. This will now be illustrated in the next paragraph.

Effect of plastic deformation due to glide on a given slip system on the orientation of the crystal axis:

We have just seen that if slip takes place on a given plane (n) along a given direction (b) the magnitude of shear can be represented by a matrix having only one nonzero term. In this case the reference frame consists of the unit vector along b and the slip plane normal n. However crystal indices are denoted with respect to the crystal reference axes. The direction cosines of the slip vector b and slip plane normal n are shown in Fig 9 along with the sketch of a single crystal. The tensile axis is given by vector l.



		Crystal axes		
		OX_1	OX_2	OX_3
b	OX_1'	b_1	b_2	b_3
n	OX_2'	n_1	n_2	n_3
	OX_3'	z_1	z_2	z_3

Fig 9: Sketch showing the orientation of a single crystal and its slip plane & slip direction (b). Unit vector n denotes slip plane normal. Crystal axes are OX_1, OX_2 & OX_3. The direction cosines of the reference axes OX_1' & OX_2' with respect to crystal axes are indicated in a tabular form.

The displacement gradient matrix with respect to the slip vector (b) as axis OX_1' and slip plane normal (n) as axis OX_2' is a 3x3 matrix with only one nonzero term. If this is represented by a matrix e_{ij}' the only

nonzero term is e_{12}'=□. This can be converted to displacement gradient with respect to the crystal reference axes using the following expression.

$$e_{ij} = a_{ki}a_{lj}e_{ij}^{'} = a_{1i}a_{2j}e_{12}^{'} \quad where\ a_{ij}\ denotes\ elements\ of\ direction\ cosine\ matrix \qquad (9)$$

Therefore the components of e_{ij} with respect to the crystal reference axes can be represented as:

$$[e_{ij}] = \begin{bmatrix} b_1n_1 & b_1n_2 & b_1n_3 \\ b_2n_1 & b_2n_2 & b_2n_3 \\ b_3n_1 & b_3n_2 & b_3n_3 \end{bmatrix} \qquad (10)$$

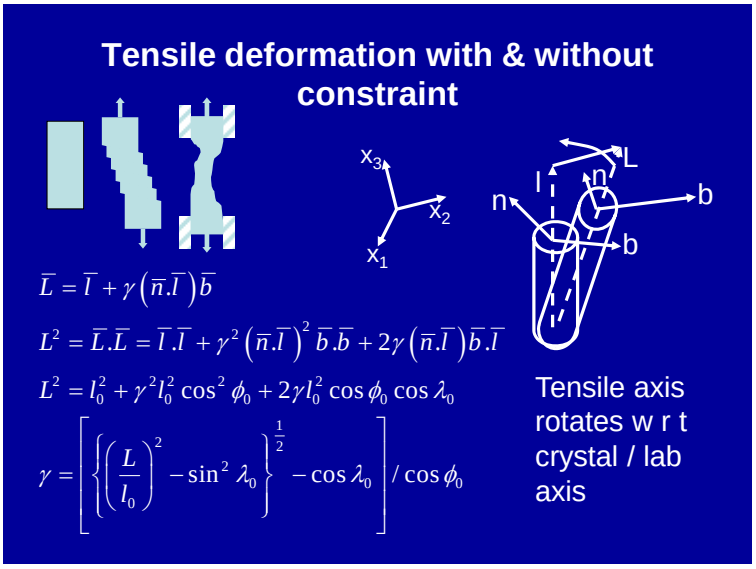
Equation 10 defines the change that occurs as a result of unit slip on one slip system. If you multiply this with the magnitude of slip (□) add a unit matrix to it you would get the transformation matrix, given in equation 11.

$$[T_{ij}] = \begin{bmatrix} 1+\gamma b_1n_1 & \gamma b_1n_2 & \gamma b_1n_3 \\ \gamma b_2n_1 & 1+\gamma b_2n_2 & \gamma b_2n_3 \\ \gamma b_3n_1 & \gamma b_3n_2 & 1+\gamma b_3n_3 \end{bmatrix} \qquad (11)$$

If [l] & [L] denote the indices of the tensile axis (vector) before and after slip of magnitude □; then the relation between two is given by equation 12.

$$\begin{bmatrix} L_1 \\ L_3 \\ L_3 \end{bmatrix} = \begin{bmatrix} 1+\gamma b_1n_1 & \gamma b_1n_2 & \gamma b_1n_3 \\ \gamma b_2n_1 & 1+\gamma b_2n_2 & \gamma b_2n_3 \\ \gamma b_3n_1 & \gamma b_3n_2 & 1+\gamma b_3n_3 \end{bmatrix} \begin{Bmatrix} l_1 \\ l_2 \\ l_3 \end{Bmatrix} \qquad (12)$$

This means that $L_i = l_i + \gamma\left(b_in_jl_j\right)$ where the repeated index j denotes summation from 1 to 3. Often this is represented as vector multiplication for brevity. The relation between the tensile axes before and after deformation can therefore be represented by the set of equations given in slide 7.



Slide 7: This displays the relation between tensile axes before and after slip. Note that the indices of the slip system described with respect to the crystal axes remain unchanged. The subscript 0 has been used to represent the values before deformation. See text for the details about the symbols.

Shear stress estimation

Slip plane remains undistorted. Its area is the same as the original area.

$$\tau = \frac{P\cos\lambda}{A_0/\cos\phi_0} = \frac{P}{A_0}\cos\lambda\cos\phi_0$$

$$\cos\lambda = \frac{\bar{L}.\bar{b}}{|L|} = \frac{\bar{l}.\bar{b} + \gamma(\bar{l}.\bar{n})\bar{b}.\bar{b}}{L}$$

$$\cos\lambda = (\cos\lambda_0 + \gamma\cos\phi_0)\frac{l_0}{L}$$

$$\tau = \left(\frac{P}{A_0}\right)\cos\phi_0\left\{1-\left(\frac{l_0\sin\lambda_0}{L}\right)^2\right\}^{\frac{1}{2}}$$

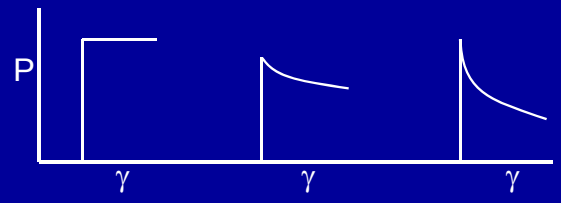
Slide 8: This displays the expressions that could be derived to show how the shear stress on the slip plane changes with increasing strain. For a given P shear stress increases as L increases.

Geometrical softening

$$\tau = \left(\frac{P}{A_0}\right)\cos\phi_0\left\{1-\left(\frac{l_0\sin\lambda_0}{L}\right)^2\right\}^{\frac{1}{2}}$$

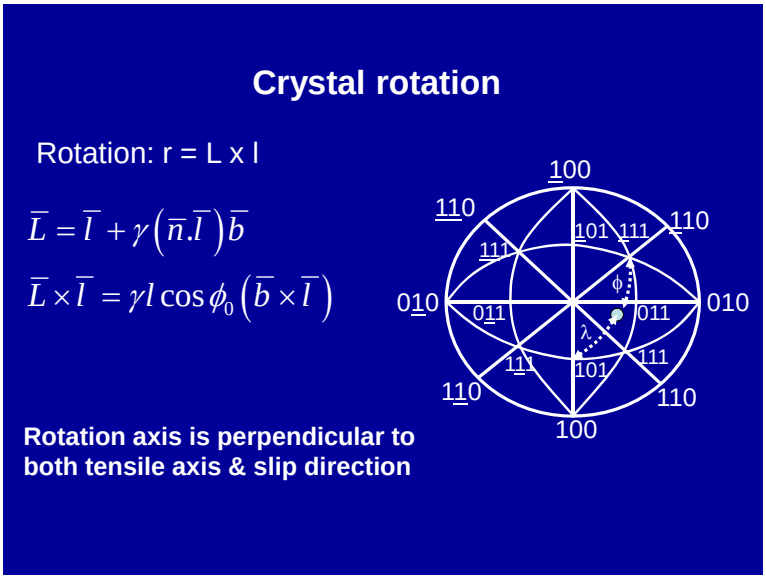
$$\frac{d\ln\tau}{dL} = \frac{l_0^2\sin^2\lambda_0}{L^2 - l_0^2\sin^2\lambda_0}$$

As L increases shear stress increases. If there is no work hardening material would exhibit softening.

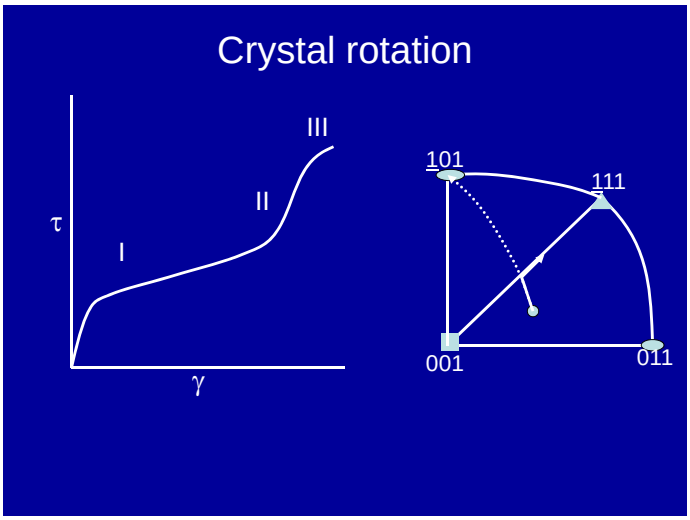


Slide 8: Illustrates the concept of geometric softening. The magnitude of softening is a function of the orientation of the tensile axis. Since the magnitude of resolved shear stress (?) increases with increasing L often the crystal may continue to deform at constant P once the CRSS is reached if there is some

amount of work hardening. In case there is no work hardening load required to sustain deformation may drop with increasing strain. This phenomenon is known as geometric softening. The sketch at the bottom of the slide shows 3 cases (i) crystal continues to deform at constant P (ii) shows a drop in the magnitude of P needed to sustain glide (iii) a case where there is significant softening.

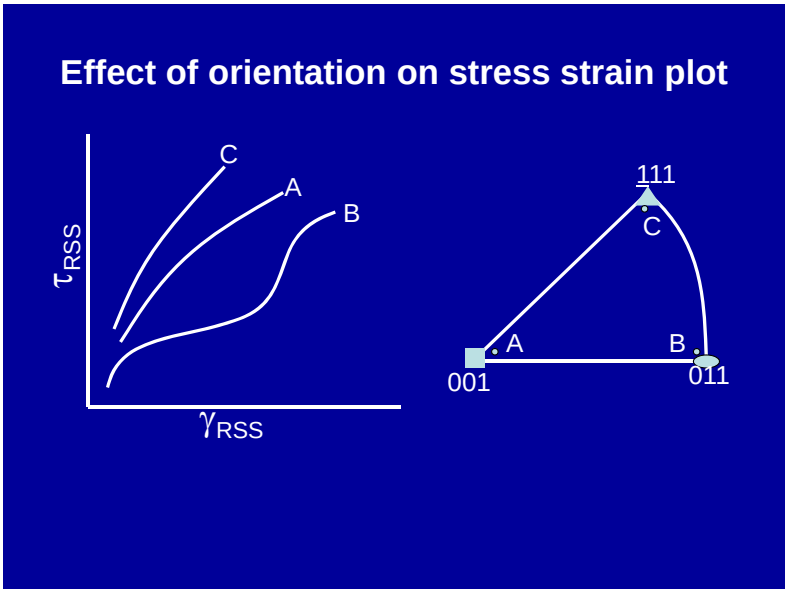


Slide 9: Illustrates how to find the indices of the axis about which the tensile axis is expected to rotate. The axis of rotation is perpendicular to both the initial and final orientations of the tensile axis. Therefore the axis of rotation should be given by the cross product of the two. It means that the axis of rotation is perpendicular to both tensile axis and slip direction.



Slide 10: This shows a typical resolved shear stress versus shear strain plot of a crystal whose tensile axis lies with the stereographic triangle. Let the indices of the axis before deformation be $[\bar{1}23]$. With deformation it would tend to move along the great circle joining the poles $[\bar{1}23]$ &$[\bar{1}01]$ as shown.

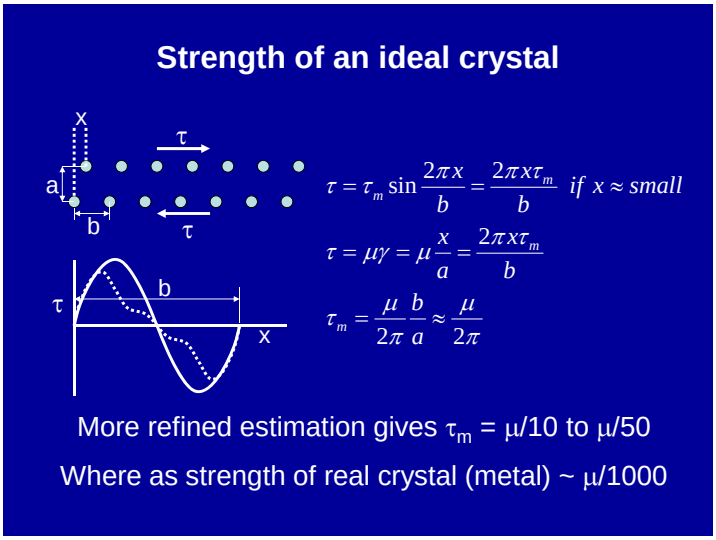
Clearly this is $[1\bar{1}1]$. As long as glide takes place only on one slip system there is little work hardening. As the tensile axis moves to point on the line (great circle) joining the poles [001] & $[\bar{1}11]$ it would start gliding on another slip system. This consists of $(\bar{1}\bar{1}1)[011]$. This stage is known as duplex slip. This is characterized by significant work hardening. Now it would tend to rotate about another axis. As a result the pole (tensile axis) would move along the great circle joining [001] & $[\bar{1}11]$.



Slide 11: Shows sketches illustrating the effect of crystal orientation on the nature of resolved shear stress versus resolved shear strain plot. The one located near <011> pole would exhibit significant period of easy glide. However this may be very short for A & C where these are very close to the line joining 001 & $[\bar{1}11]$.

What is the strength of an ideal crystal?

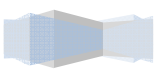
We have just seen how extensive plastic deformation could take place in crystals by glide. This happens on application of shear stress higher than its CRSS. Since atoms in a crystal are arranged in a periodic fashion as these are forced to move away from their mean position of rest there will be opposing restoring force acting on the atoms. In view of the periodic arrangement of atoms the restoring shear stress may be represented as a sine function. The following slide illustrates how with this simple assumption it is possible to derive an expression an expression for the shear strength of an ideal crystal.

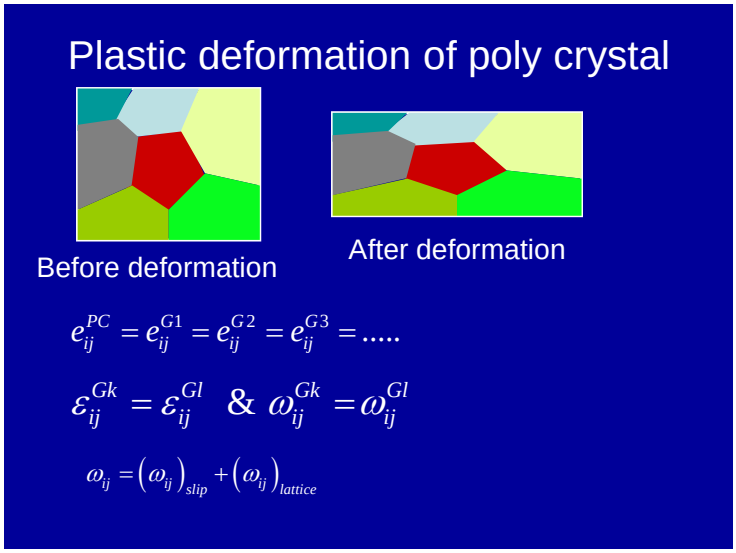


Slide 12: The top figure shows the location of atoms on either sides of the slip plane displaced by a small distance x. Note that <a> denotes the distance between the planes and <b> is distance between two nearest atoms on the slip plane. The restoring stress (□) has been represented by a periodic sine function (see the first equation). The second figure gives a graphical representation of how □ varies with displacement. Note that □ is 0 at three points 0, b/2 & b. The maximum restoring shear stress is □$_m$. This occurs at x=b/4. Let us try to relate this to the magnitude of shear modulus. Assume x to be very small. The first equation gives an approximate magnitude of □□as a function of x. Shear stress is directly proportional to shear strain. In this case shear strain is x/a. Thus an alternative expression for □ has been obtained. See the second equation. Equating the two an expression for □$_m$ has been derived. Note the last equation. Since $b \approx a$ the maximum shear strength of the crystal is given by □/2π. More refined calculations using better description of the periodic nature of the restoring stress as shown by the dotted line is possible. Still the estimate remains two orders of magnitude lower than the of the real strength of crystal.

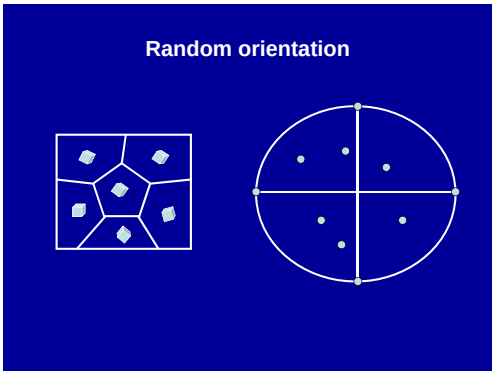
Deformation of poly crystalline material:

Metals we use are rarely made of single crystal. They consist of numerous grains (crystals) arranged at random. This means that the orientation of a particular grain is independent of its neighbors. These are separated by boundaries. Across the boundary there is no relation between the atomic arrays. However when deformation occurs there must be continuity all along the boundaries. This is possible if all the components of deformations of all the grains are identical. This is schematically shown in slide 13.

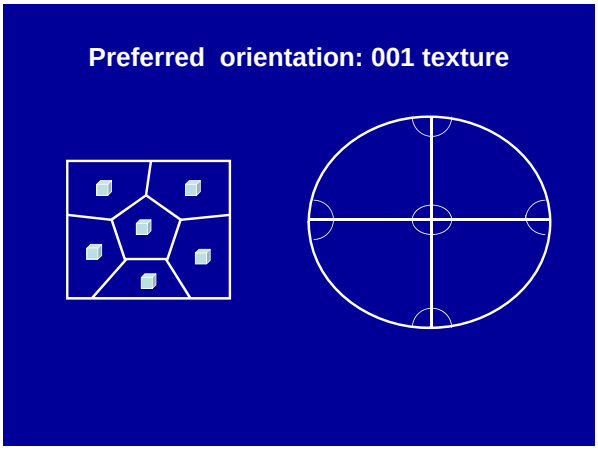




Slide 13: Shows schematic microstructures of polycrystalline metal before and after plastic deformation. Note that the strains in each grain are identical. There is continuity across the boundary. This is possible when all the components of strains in individual grains are identical. It can be expressed mathematically by the sets of equations given above. Note that e_{ij}^{PC} is one of the components of deformation (change in shape) of the polycrystalline metal. The symbol e_{ij}^{G1} denotes components of deformation of grain <G1>. Recall that deformation due to slip is made of strain and rotation. In matrix form it is written as $e_{ij}^{Gk} = \varepsilon_{ij}^{Gk} + \omega_{ij}^{Gk}$ for the kth grain of the metal. The rotation too has two components. One we have discussed is associated with slip. Apart from this there can be lattice rotation of individual grains. Additional modes of deformation are needed to maintain continuity. Grain rotation and sliding are some of these.



Slide 14: Shows a schematic microstructure of a polycrystalline sample. The cube in each grain denotes its orientation. The stereographic projection that is shown alongside displays positions of cube poles as points. They are scattered randomly. In this case cubes poles of only a few grains are shown. If there are n such grains each will have 3 cube poles. The number of such points will be 3n.



Slide 15: This shows a case where the grains are so oriented that the cubes planes are parallel to the top surface. Note the way the cubes are placed within the grains. The figure on the right is a stereographic projection of the sample displaying the locations of cube poles. These are located within the small circles shown on the stereographic projection. Such projections displaying locations of specific poles are known as pole figures.

The constraints mentioned in slide 13 require that slip in each grain must take place simultaneously on 5 slip systems. This is why the stress strain diagram of a polycrystalline metal does not exhibit stages of easy glide or duplex slip as in a single crystal. The deformation results in changes in the shapes of grains and their orientations.

Summary

In this module the main features of plastic deformation have been explained. Metals we use are mostly polycrystalline. Its deformation behavior therefore depends not only on its crystal structure but also on the ways these arranged in the metal. Deformation in a crystal takes place by slip and twin. Both occur due to shear on specific planes along specific directions. The basic difference between the two has been brought out. Importance of multiple slip to allow arbitrary deformation in crystals has been explained. An attempt has been made to give some amount of quantitative insight to help compute the change in the orientation of a crystal due to plastic deformation using mathematical techniques.

Exercise:

1. In which mode of plastic deformation atomic displacement could be less than inter atomic spacing?
2. Estimate the magnitude of shear strain for (111) $[11\bar{2}]$ twin in fcc lattice.
3. What is the effect of tensile stress on lattice spacing?
4. Show schematic resolved shear stress versus shear strain diagrams of fcc crystal if the tensile axes were (a) [123] (b) [001]
5. What is the difference between simple shear & pure shear? Under which category will you place plastic deformation by slip?
6. What is the effect of plastic deformation on lattice parameter?

7. Draw a standard [001] projection showing all possible slip planes & directions for a bcc crystal. Assume slip can take place only on {110} planes.
8. When does a polycrystalline material have same yield strength along all possible direction?
9. Estimate the ideal cleavage strength and shear strength of pure iron. Given E = 211 GPa and G = 83 GPa.

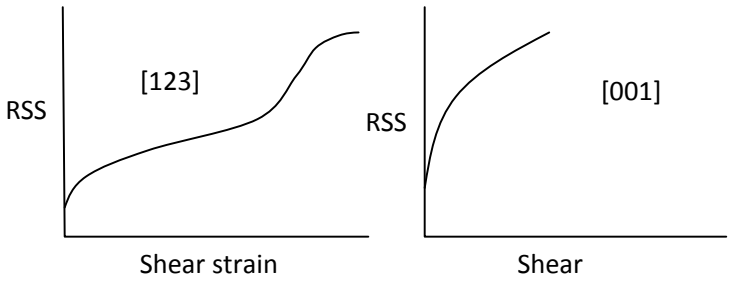
Answer:

1. Twin.

2. The distance between twin plane = $\frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{a}{\sqrt{3}}$ and the magnitude of slip $=\frac{a}{6}[11\bar{2}] = \frac{a\sqrt{1^2+1^2+(-2^2)}}{6} = \frac{a}{\sqrt{6}}$ (this represents the distance between two atoms along$[11\bar{2}]$). Shear strain is the ratio of magnitude of slip to the distance between the two planes $= \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}} = 0.71$

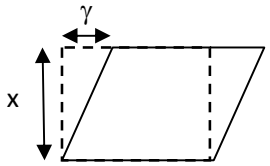
3. Lattice spacing increases with tensile stress till it reaches its elastic limit. Elastic strain is equal to the ratio of change in lattice spacing along the tensile axis to its original value $=\left(\frac{\Delta d}{d}\right)$.

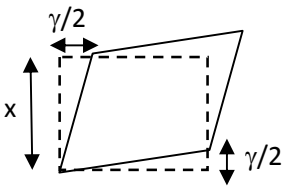
4.



In case of [123] initially resolved shear stress reaches its critical value for a single slip system. Therefore all 3 stages of deformation are seen. In case of [001] RSS reaches CRSS simultaneously on several slip system (8 to be precise). Therefore 3 distinct stages are not seen.

5. Simple shear represents displacement or slip on a plane along a specified direction. This is schematically represented as follow:

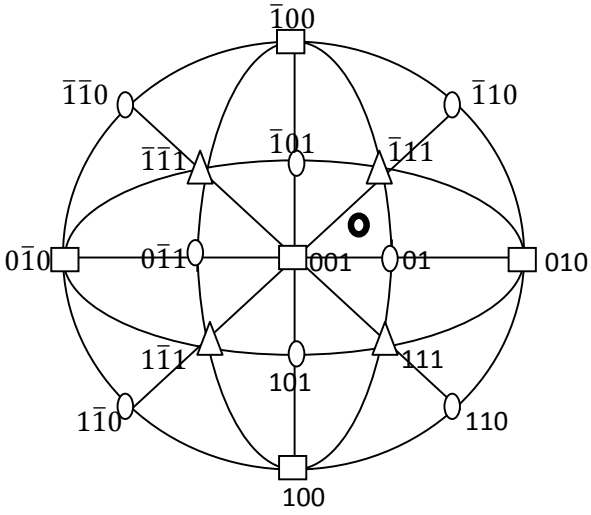




Simple shear: x is normal to plane on which shear has taken place & y is displacement. Displacement gradient is: $e_{xy} = \frac{y}{x} = \gamma$. Using the notation used it is e_{12}. All other components are zero. The matrix is not symmetric. Slip is a simple shear

Pure shear: simple shear (e_{ij}) is equal to sum of pure shear (ε_{ij}) and rotation (ω_{ij}) as shown above. This makes the strain matrix symmetric. $e_{ij} = \epsilon_{ij} + \omega_{ij}$

$$\begin{bmatrix} 0 & \gamma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \gamma/2 & 0 \\ \gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \gamma/2 & 0 \\ -\gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

6. Plastic deformation does not alter crystal structure or its dimension. Lattice parameter after deformation is still the same.
7. 

Slip systems of bcc crystal are {101}< $\bar{1}11$ >There are 12 such system. These are shown in standard projection. For a pole lying within a stereographic triangle slip plane & direction having maximum resolved shear stress lie in the adjacent triangle. This is shown for one case where the pole is marked as a circle. The slip plane is $(\bar{1}01)$ and slip direction is [111].

8. Strength of a crystal may vary with the direction of loading. Polycrystalline metals may have a large number of grains. Its properties will be a function of the entire group. If these are randomly oriented then one would expect its properties to be isotropic.

9. Ideal shear strength $\tau_{max} = \frac{G}{2\pi} = \frac{83}{2\pi} = 13.21\ GPa$. For estimating cleavage strength please see problem 15 of chapter 1. This gives $\sigma_{max} = \sqrt{\frac{E\gamma}{a_0}}$ & $\gamma = \frac{a^2}{\pi^2}\frac{E}{a_0} \approx \frac{Ea_0}{10}$ since ***a*** is nearly equal to a_0. Thus $\sigma_{max} = \sqrt{\frac{E\gamma}{a_0}} \approx \frac{E}{\sqrt{10}} = \frac{211}{3.16} = 67\ GPa$. These are nearly two orders of magnitude higher than the real strength of iron.