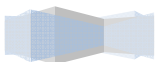


Module 32

Heat treatment of steel II

Lecture 32

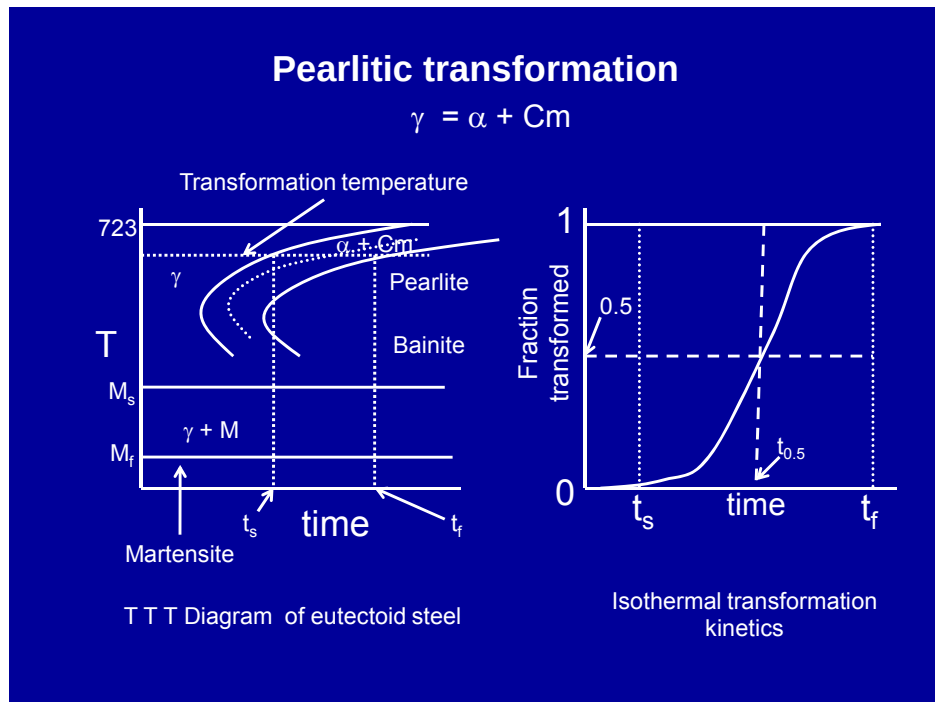
Heat treatment of steel II



Keywords : Kinetics of pearlitic transformation, Johnson – Mehl –Avrami equation, effect of carbon content on T T T diagram, bainite: low temperature eutectoid transformation, characteristics of martensitic transformation, Bain distortion, morphology of martensite, effect of %C on hardness and lattice parameter of martensite

Introduction

We are by now familiar with three different types of transformation in eutectoid steel. These are pearlitic, bainitic and martensitic transformations. The first two are the products of isothermal transformation where diffusion plays a major role. Both pearlite and bainite are made of ferrite and carbide. The difference lies in the shape, size and distribution of carbides. Pearlite is made of alternate layers of cementite and ferrite plates. This is often described as a lamellar structure. The spacing between the plates of ferrite and cementite is known as inter lamellar spacing. Pearlite that forms at higher temperature has a relatively large inter lamellar spacing in comparison to the one that forms near the nose of the TTT diagram. The product of isothermal transformation below the knee of the TTT diagram is known as bainite. It has two variants one that forms just below the knee of the TTT diagram is called feathery or upper bainite. It is an intimate mixture of ferrite and a fine dispersion of tiny platelets of cementite. The low temperature version of bainite consists of a fine dispersion of a different type of carbide known as epsilon carbide ($\text{Fe}_{2.4}\text{C}$) in ferrite matrix. The transformation in both cases occurs by nucleation and growth. At a given temperature such a process has a set of characteristic time at which the transformation begins (t_s) and a time at which it is complete (t_f). If you monitor fraction transformed at a given temperature and plot the same against time you get an 'S' shaped plot (see slide 1).



Slide 1

The fraction transformed is zero until $t = t_s$. Thereafter it increases until it reaches 1 at t_f . We also looked at the equation which best describes the evolution of f as a function of t . It is reproduced once again:

$$f = 1 - \exp\left\{-\left(\frac{t}{\tau}\right)^n\right\} \quad (1)$$

It uses two parameters (n & τ) which can be estimated by regression analysis of the experimental data. It was also shown how the isothermal transformation can be modeled based on a set of very simple assumptions. For example it was shown that if the pearlite nodules grow at a constant rate (G mm/sec) after N numbers of stable nuclei have formed, the exponent n is equal to 3 whereas τ the characteristic time is a function of N & G . Let us begin this lecture by deriving an expression for f with a different set of assumptions.

Kinetics of Pearlitic transformation:

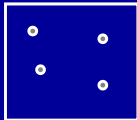
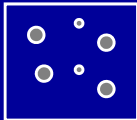
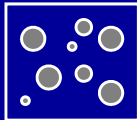
Let us assume that during the isothermal transformation of super cooled austenite both the nucleation and the growth rates are constant. $\dot{N}$ = the rate of nucleation in number /sec /unit volume and $\dot{G}$ = growth rate in mm /sec. Slide 2 shows a schematic representation of the transformation process by a set of sketches. Note that with time both the number of pearlite nodules and the radius keep increasing. At any instant t the radius of a nodule is equal to $\dot{G}t$. The increase in extended volume (df_{ext}) is given by the average volume of nodules times the number of nuclei that forms with an infinitesimal time (dt) which is equal to $\dot{N}dt$. It is also equal

to $\frac{df}{1-f}$. On integration you get an expression for f as a function of time (see slide 2). Note that in this case the exponent $n = 4$. The characteristic time is a function of both nucleation & growth rates. Slide 2 gives the exact expression for τ . The isothermal growth of pearlite nodules in steel is best described by the equation for f given in slide 2. Often it is referred to as Johnson - Mehl -Avrami equation. Note that the magnitude of n is different from that in the case of the site saturation model. This shows that n is an indicator of the mechanism of transformation.

Johnson - Mehl – Avrami EQN

Assumption: constant nucleation & growth rate

Constant nucleation
& growth rate

$$df_{ext} = \frac{4\pi}{3} (\dot{G}t)^3 \dot{N}dt = \frac{df}{1-f}$$

$$f = 1 - \exp\left(-\frac{\pi}{3} \dot{N} \dot{G}^3 t^4\right) = 1 - \exp\left\{-\left(\frac{t}{\tau}\right)^4\right\}$$

$$\tau = \left(\frac{\pi}{3} \dot{N} \dot{G}^3\right)^{1/4} \quad n: \text{indicator of mechanism}$$

Slide 2

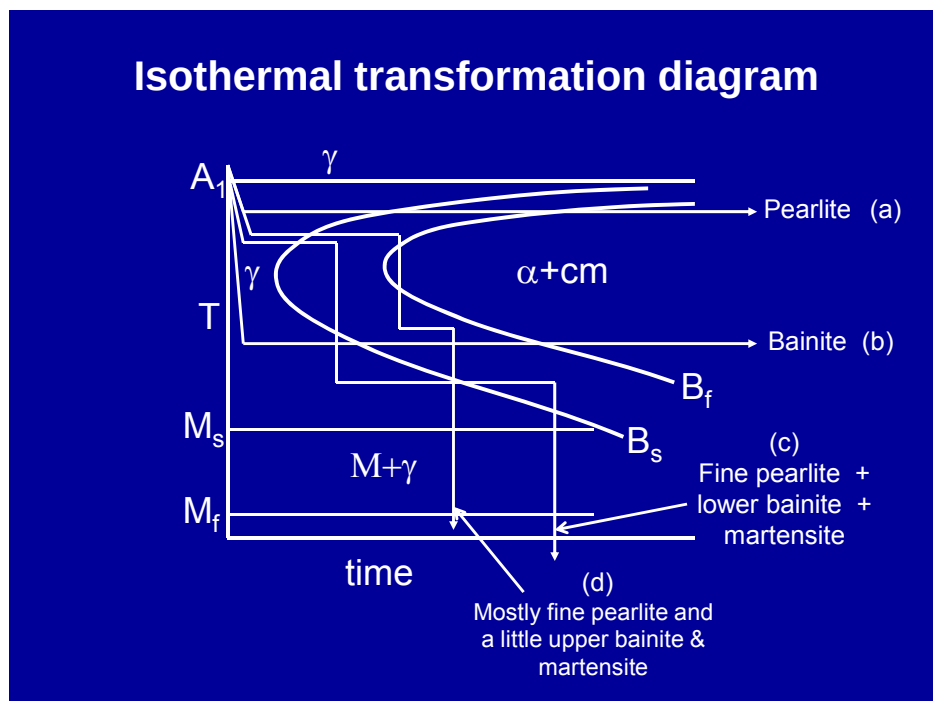
The expression for the fraction transformed (f) does explain the 'S' shape of the f versus time (t) plot at a given temperature. It was derived with the assumption that the rates of nucleation and growth are constant. It is valid for isothermal transformation. The expression can be used to estimate the time needed for 50% transformation. It can be shown by substituting $f = 0.5$ in equation 1 that it is given by:

$$t_{0.5} = 0.9\tau \approx \frac{0.9}{\dot{N}^{1/4} \dot{G}^{3/4}} \quad (2)$$

Let us look at the effect of temperature on the time needed for 50% transformation to take place. It is inversely proportional to the rate of nucleation and the rate of growth. If the temperature is close to the critical temperature the driving force for nucleation is negligible because the degree of super cooling ΔT is low. The rate of growth of nodules depends on the diffusivity of carbon in austenite. It is high at higher temperatures. At low ΔT although the growth rate is high the product $\dot{N}^{1/4} \dot{G}^{3/4}$ is dominated by negligible rate of nucleation (approaching zero). On the other hand at high ΔT the product $\dot{N}^{1/4} \dot{G}^{3/4}$ is dominated by a very

small rate of growth. The magnitude of product $\dot{N}^{1/4} \dot{G}^{3/4}$ in both the cases is low therefore $t_{0.5}$ is very high. At an intermediate temperature where the two terms of the product are nearly the same ($\dot{N}^{1/4} \approx \dot{G}^{3/4}$) it is likely to have the highest value. This corresponds to the lowest value for $t_{0.5}$. This is the temperature where the rate of transformation is the highest. This suggests that initially $t_{0.5}$ decreases as the temperature of transformation decreases. It reaches a minimum at an intermediate temperature and thereafter it starts increasing. If it is plotted along the horizontal (x) axis against temperature along the vertical (y) axis you would get a C shaped plot.

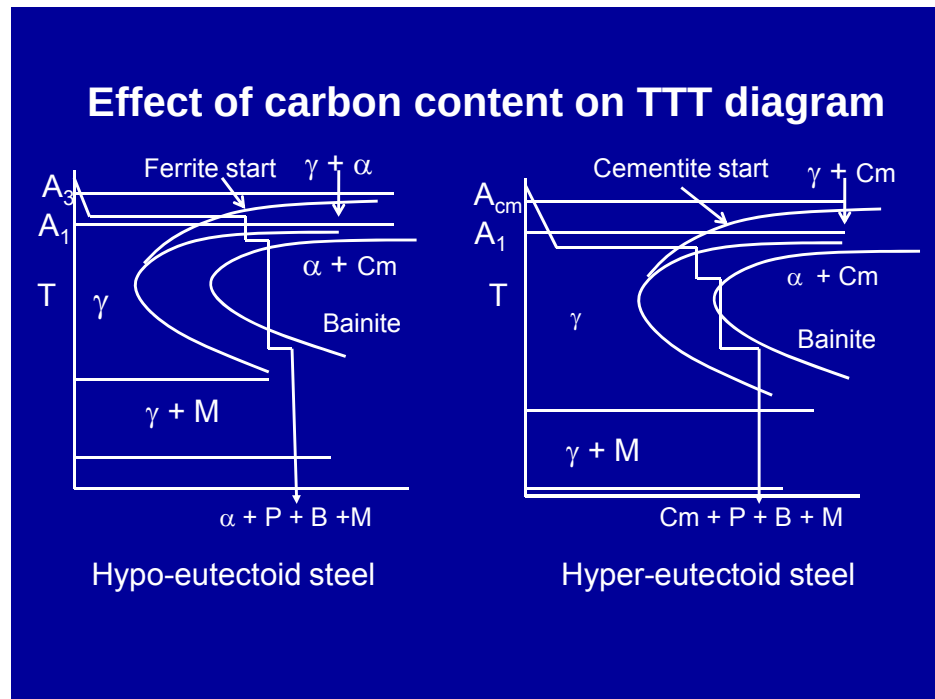
Effect of carbon content on the T T T diagram of steel:



Slide 3

Isothermal or time temperature transformation diagram gives at a glance the variety of structures that can be developed in steel by proper choice of cooling schemes. Slide 3 presents such a diagram for eutectoid steel. It has a few cooling schemes super imposed on the diagram. The scheme (a) & (b) allows transformation to go to completion at the same temperature. The final product is 100% pearlite in (a) whereas it is 100% bainite in (b). It is also possible to have schemes where there are multiple isothermal steps. The first isothermal hold in scheme (c) allows the steel to transform partly into fine pearlite, it is then quenched to a temperature where it partially transforms into lower bainite and finally it is quenched to room temperature when the remaining austenite transforms into martensite. The final structure is a

mixture of fine pearlite, lower bainite and martensite. The final structure in scheme (d) would consist of mostly fine pearlite and a little amount of upper bainite & martensite.



Slide 4

Slide 4 gives TTT diagrams of hypo & hyper eutectoid steel. It gives the main features of isothermal transformation when the steel is cooled from austenitic state. In the case of hypo eutectoid steel the usual austenitization temperature is 30°- 40°C above the upper critical temperature (A_3) whereas in the case hyper eutectoid steel austenitization temperature is 30°- 40°C above A_{cm} . Note that in these diagrams there is an additional domain where precipitation of pro-eutectoid phase from austenite takes place. This may occur at a temperature lower than that of A_3 or A_{cm} . The line labeled ferrite start in the diagram for hypo eutectoid steel denotes the time at which the precipitation of ferrite begins. It is accompanied by an increase in %C of the remaining austenite. When it reaches eutectoid composition; the transformation of the balance austenite to pearlite starts. The rest of the features of the transformation diagram for hypo-eutectoid steel are exactly same as those of the eutectoid steel.

Slide 4 also gives the TTT diagram of hyper-eutectoid steel. In this case precipitation of pro-eutectoid cementite takes place at a temperature lower than that of A_{cm} . The time at which this begins is shown by the line labeled 'cementite start'. %C in the balance austenite decreases with the precipitation of cementite. It continues till it reaches the eutectoid composition. The main features of the subsequent transformation are exactly same as those of the eutectoid steel. It appears from these diagrams that hypo or hyper eutectoid steel could be transformed into 100% pearlitic structure if the transformation takes place near the nose of the TTT

diagram. It suggests that the amount of pro-eutectoid phase found in the steel is a function of the temperature of transformation.

You may select or plan suitable schemes of cooling to get a variety of microstructures in steel. This is illustrated by superimposing cooling curves on the two TTT diagrams given in slide 4. These are isothermal transformation diagrams. It can correctly predict transformations that depend on diffusion only during isothermal hold. Therefore the cooling curves are made of several steps indicating iso-thermal hold. The change in temperature from one isothermal hold to another is shown by vertical lines to represent extremely fast cooling to stop diffusion controlled transformation. However austenite can also transform into martensite. The process is athermal. It cannot be suppressed even by extremely fast cooling. It occurs below a temperature called M_s as long as it continues to fall until M_f the transformation is nearly complete. Note that this type of transformation can take place only within a specific range of temperature. The cooling schedule shown on the TTT diagram of hypo-eutectoid steel is likely to give a microstructure consisting of pro-eutectoid ferrite, pearlite, bainite and martensite. Similarly the cooling schedule shown on the TTT diagram of hyper-eutectoid steel is likely to give a micro-structure consisting of pro-eutectoid cementite, pearlite, bainite and martensite.

Let us look at the effect of %C on its TTT diagram. In the case of hypo-eutectoid steel the temperatures denoted as A_3 , M_s & M_f would decrease with an increase in %C. However A_1 , the eutectoid temperature remains unchanged. The rest of the curves denoting, the start of ferrite precipitation, the onset of pearlitic or bainitic transformation and those denoting the completion of pearlitic or bainitic transformation, shift towards the right of the diagram. In other words the diffusion controlled transformations need longer incubation period and longer time for completion as %C increases. The effect of carbon %C on the locations of the various lines & curves in the TTT diagram of hyper-eutectoid steel is very much the same except that the line denoting A_{cm} increases with increasing %C. (This is where the trend differs from that of the hypo-eutectoid steel). The line A_1 is independent of % C. The M_s and M_f temperatures decrease as %C increases. In high carbon steel, M_f temperature may even be lower than that of the room temperature. Such steels on quenching to room temperature would have some amount of austenite. It needs further cooling below room temperature (even below 0°C) so that transformation of austenite to martensite is nearly complete. The rest of the curves representing the onset of completion of diffusion controlled transformation shifts towards higher times as %C increases. This trend is exactly same as that in hypo-eutectoid steel. Apart from %C there are several other factors that affect the locations of various lines and curves of TTT diagram. These are austenitization temperature and the presence of alloying element & inclusions in the steel. We shall talk about it in a subsequent module.

Bainite: low temperature eutectoid:

Bainite forms in steel if it is quenched from the state of austenite to a temperature below the nose of the TTT diagram but above the M_s (temperature) and held there long enough for the transformation to go to completion (see fig 1). It consists of an intimate mixture of ferrite and extremely fine carbides. Unlike pearlitic transformation it starts with the nucleation of ferrite laths or plates within which the tiny particles of carbides are dispersed. The precipitation occurs simultaneously as in the case of pearlitic transformation. It can therefore be viewed as a product of low temperature eutectoid transformation. Like pearlite the kinetics of bainitic transformation too is controlled by diffusion. However the nucleation of bainite laths or plates require higher degree of super-cooling.

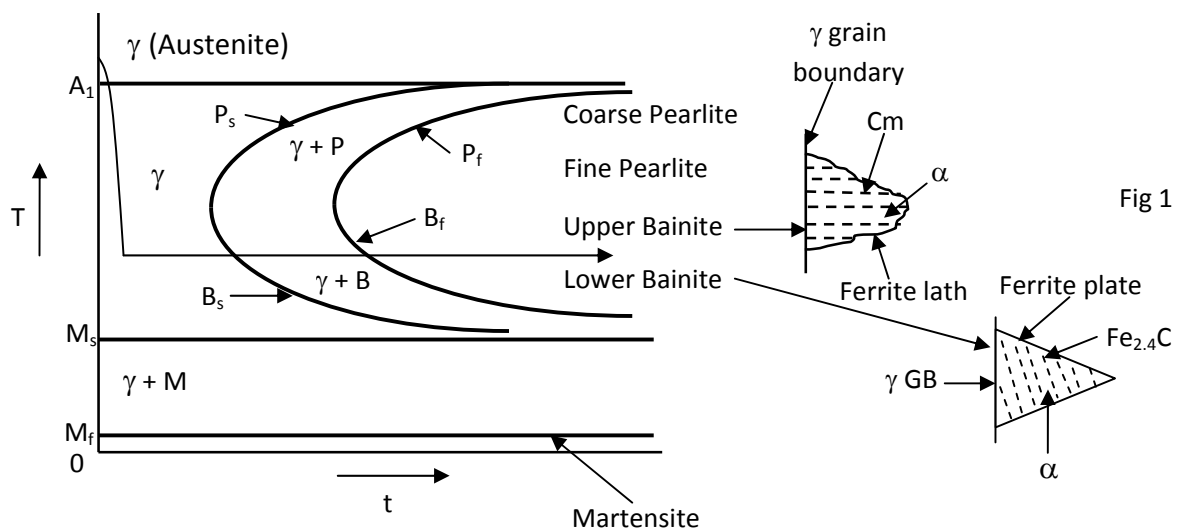


Figure 1 shows the TTT diagram of eutectoid steel. It is reproduced here to show the main features of bainitic transformation. Bainite that forms near the nose of the TTT diagram is known as upper or feathery bainite. It forms with the nucleation of ferrite laths on specific crystallographic planes of austenite and carbide platelets form on the boundary of the lath as it keeps extending. This leads to a microstructure as shown in fig 1. The gap between the platelets is too short to be seen under optical microscope. There is a close similarity between the structure of fine pearlite and upper bainite. The main difference is in the size of the cementite plate. In pearlite, cementite plates are long whereas in bainite it is made of a large number of broken platelets aligned along the axis of the ferrite lath.

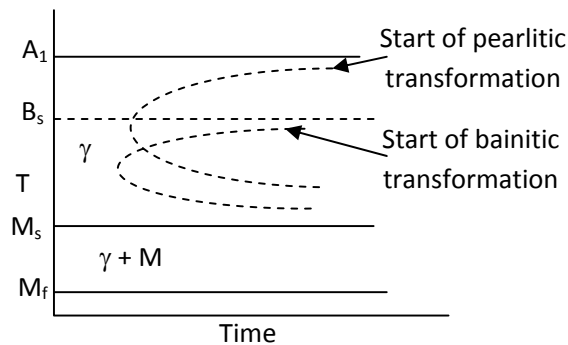


Fig 2

The time it takes for pearlitic or bainitic transformations to start can be represented by two different 'C' shaped plots. It is shown in fig 2. Pearlitic transformation occurs just below the eutectoid temperature (A_1). B_s is the temperature below which austenite can transform into bainite. There should be a pair of similar 'C' shaped plots representing the time it takes for the transformations to be completed. These have not been shown to avoid overcrowding of too many curves within the sketch in fig 2. The lower bainite has a distinctly different structure. Its main features are shown with the help of a sketch in fig 1. Lower bainite, also known as acicular bainite, forms with the nucleation of ferrite plates with a fine dispersion of thin rod shaped carbides. The composition of the carbide corresponds to $\text{Fe}_{2.4}\text{C}$. It is known as epsilon carbide (ϵ). These are aligned at an angle with respect to the axis of the ferrite plate. We would later come across similar structure having a dispersion of epsilon carbide in a ferrite matrix during tempering of steel. This is why some time lower bainite might get identified as tempered martensite. This can be avoided by a careful examination under TEM or SEM at higher magnification. Bainite is also characterized by the shorter distance between neighboring carbides. This suggests that diffusion of carbon during bainitic transformation is limited over a very short distance. We know that the diffusion distance is given by $\sqrt{Dt}$ where D denotes diffusivity of carbon in austenite. Bainite forms at a relatively low temperature where the diffusivity of carbon is low. This is why carbides in bainite are in the forms of short platelets or thin & short rods.

Martensitic transformation:

Martensite forms in steel if it is quenched from the austenitic state to a temperature below M_s . Unlike pearlite and bainite it is a homogeneous (single) phase. Austenite is FCC. It is stable at high temperature. The solubility of carbon in austenite is much more than that in ferrite the low temperature form of iron which is BCC. If the steel is cooled slowly carbon gets enough time to

diffuse within austenite. As a result it transforms into a mixture of ferrite and carbide. However on quenching atoms of carbon are retained within the lattice even if the crystal structure transforms from FCC to BCC. The presence of excess carbon in BCC lattice is responsible for tetragonal distortion. This is why the crystal structure of martensite is BCT. It may be considered as a super saturated solid solution of carbon in alpha iron (α'). The most important characteristics of martensitic transformation are as follows:

- Athermal: Transformation occurs as long as the temperature keeps dropping.
- Diffusion-less: The composition of martensite is exactly same as that of austenite.
- Crystal structure: Body centered tetragonal. Its c/a ratio is a function of carbon content.
- Hardness: It is hard & brittle. Hardness increases with %C. It reaches R_c 64 at %C = 0.6
- Morphology: Depends on %C. It is acicular or needle like if %C > 0.3. Lath if %C < 0.3.
- Speed: Extremely fast ($\sim$ velocity of sound / elastic wave). Takes place by shear.

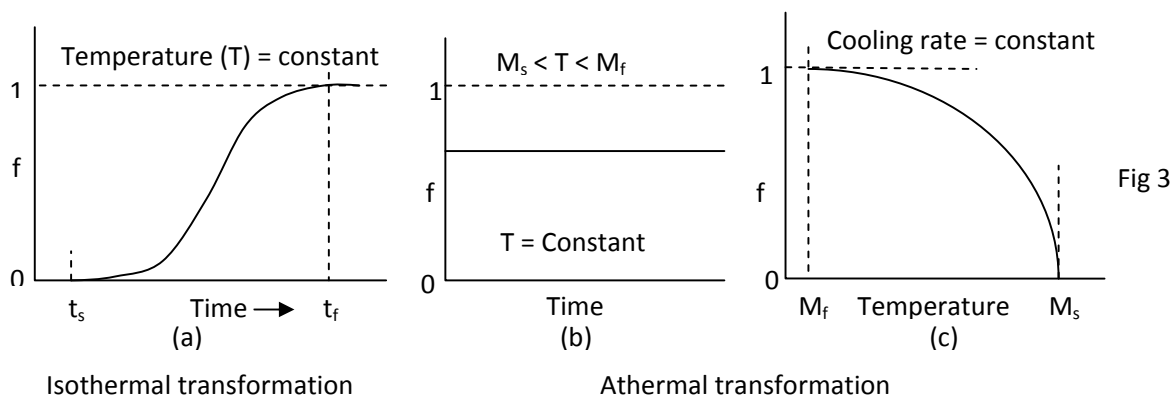
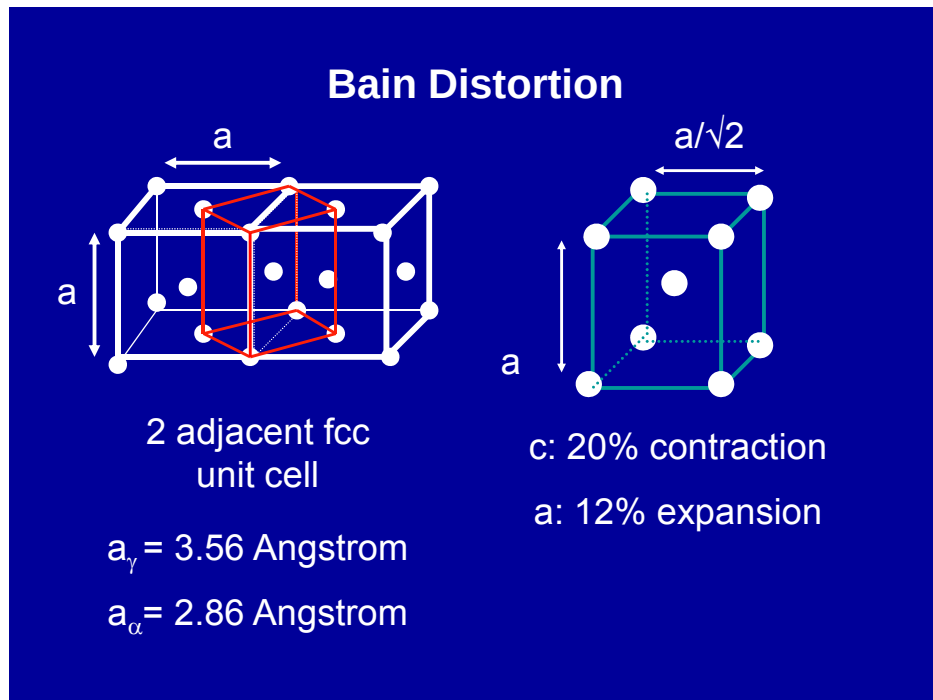


Figure 3 shows the main difference between isothermal and athermal transformation with the help of a set of plots. Figure 3(a) shows how at a given temperature f , the fraction transformed increases with time at a constant temperature. Note it has a characteristic 'S' shape. It starts after a lapse of time t_s known as the incubation period and it can go to completion at the same temperature at time t_f . These are the main features of an isothermal transformation. An athermal transformation (unlike isothermal transformation) is independent of time. This is shown in fig 3(b). For example if a sample of steel is quenched from the austenitic state (760°C for 0.8%C steel) to a temperature between M_s and M_f the transformation occurs instantaneously. Thereafter f remains constant. Figure 3(c) shows how f increases with decreasing temperature. Note that the transformation starts at M_s . The fraction transformed, f , increases from 0 at M_s to nearly 1 at M_f . In other words it continues within a specified range of temperature as long as the temperature keeps dropping. It stops if the temperature is held constant at a temperature within this range but it cannot be suppressed even by extremely fast cooling. Diffusion or the movement of solute (carbon in the case of

steel) under a concentration gradient is a time dependent process. Since athermal transformation is independent of time, diffusion cannot take place. The composition of steel remains unchanged even after the transformation. This is why martensitic transformation is also known as diffusion-less transformation.



Slide 5

Martensitic transformation is associated with a change in crystal structure from FCC (austenite) to BCT. The c/a ratio of martensite is a function of %C. It approaches 1 as %C tends towards 0. In such a situation the crystal structure of martensite can be assumed to be BCC. Slide 5 shows the implication of the change in the crystal structure from FCC to BCC. The sketch on the left shows how the atoms are located in two adjacent FCC unit cells. Note that it can also be visualized as a BCT structure with a (c/a) ratio of 1.414. The construction of this unit cell is shown with red lines. The sketch on the right is a copy of the same. The dimensions of the edges of the BCT unit cell are given in terms of the lattice parameter ' a ' of the FCC unit cell. The conversion of such a unit cell from FCC to BCC ($c/a = 1$) would amount to a contraction along the ' c ' axis and / or expansion along the ' a ' axis. The lattice parameter of austenite (FCC) is 3.56Angstrom and that of ferrite (BCC) is 2.86Angstrom. Using these it can be shown that the transformation is accompanied by around 20% contraction along the c axis and 12% expansion along the ' a ' axis. Figure 5 shows that in the presence of carbon atom there a tetragonal distortion. The resulting structure becomes BCT (Body Centered Tetragonal). The extent of distortion is a function of the number of sites occupied by carbon atoms. It increases with the concentration of carbon in the solid solution.

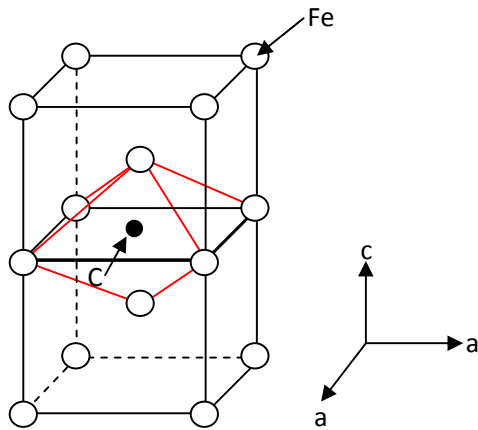
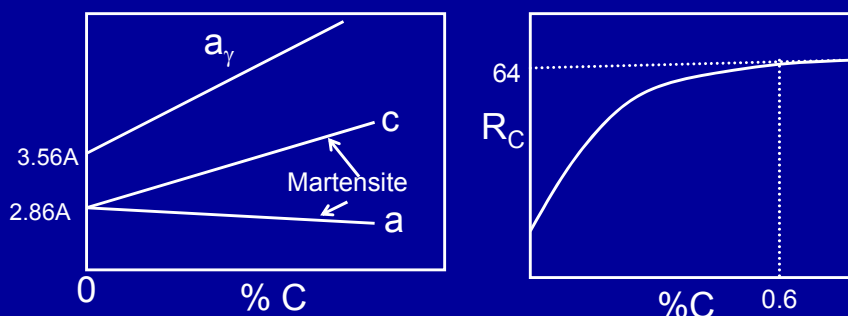


Fig 4: Shows what happens during the transformation from FCC to BCC in the presence of an atom of carbon in one of the octahedral sites. In a BCC lattice the gap between two neighboring atoms of iron along the 'c' axis is less than those between the two nearest atoms on the plane perpendicular to the 'c' axis. In the presence of a carbon atom in the interstices the iron atoms are pushed apart along the 'c' axis and those on the plane normal to it, come a little closer resulting in a tetragonal distortion. It is also known as Bain distortion.

Effect of carbon on lattice parameter & hardness

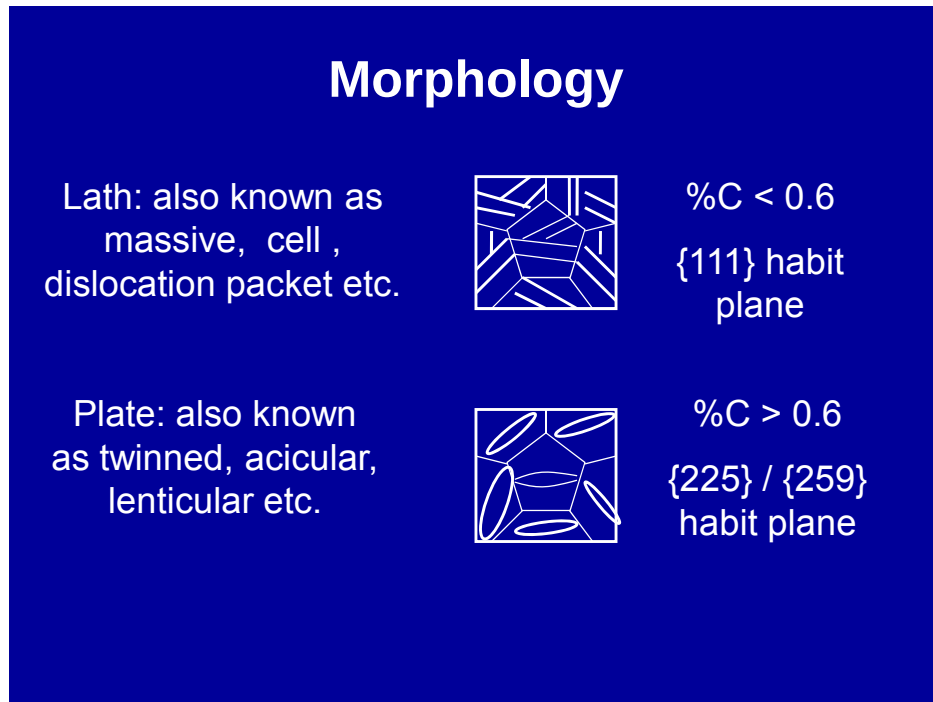


Higher carbon ~ more lattice distortion (strain) hence higher the strength

Slide 6

Slide 6 shows the effect of %C in martensite on its lattice parameter. The crystal structure of extremely low carbon martensite is BCC. The presence of excess carbon leads to tetragonal distortion. It is associated with lattice strain. It has both hydrostatic and shear stress field. It can interact and block the movements of both edge and screw dislocations. The higher the concentration of carbon atoms; the stronger is the interaction between solute atoms and dislocations. This is primarily responsible for the higher hardness of martensite. Note the trend of the hardness versus %C plot given in slide 5. The hardness in Rockwell C scale increases till 0.6% C. Beyond this there is hardly any increase in hardness. The maximum hardness in

Rockwell scale is R_c64. If the hardness of martensite is measured in Vickers scale it continues to increase even beyond 0.6%C. The harden-able grades of steel are used as tools & dies. For such applications requirements of hardness are usually specified in Rockwell 'C' scale.



Slide 7

Martensitic transformation takes place by shear. The speed at which it occurs is of the order of the velocity of sound in the medium. It emits acoustic signal which can be picked up and amplified to make it loud enough to be audible. Slide 7 shows with the help of sketches the morphology of martensite. It depends on %C. If it is less than 0.6 martensite forms as laths on specific habit planes {111} in austenite grains. It divides the original grain into several tiny cells. The cell boundary consists of dislocations. If % C is more than 0.6% martensite forms as twins within austenite grains. It has acicular (also called lenticular or needle like) shape. Nowadays it is more commonly known as plate martensite. It forms on specific planes of austenite.

Similarities between bainitic and martensitic transformation:

Bainitic transformation in steel has some feature that are common with pearlitic transformation and some features that are like those in martensitic transformation. It involves re-distribution of carbon as in pearlitic transformation and crystallographic change which is similar to that in martensitic transformation. Martensite laths or plates form in austenite through shear or twinning. This occurs on planes that remain undistorted and un-rotated. It is called the habit plane or invariant plane. The strain associated with twinning is very large. It is equal to $1/\sqrt{2}$. However invariant plane strain or the strain energy associated with it can be

minimized if the product phase is very thin. This is how ferrite plates in bainite or martensite plates form within austenite. The former needs redistribution of carbon so that the ferrite plates could grow whereas the latter does not need any change in composition. This is why the growth of martensite lath or plate is very fast whereas those in bainite is slow. If such a transformation is allowed to take place in a sample whose surface has been previously polished it results in an uneven surface. This is a characteristic of both bainitic and martensitic transformation. This is illustrated with the help of a set of sketches in fig 5. Since martensitic transformation does not involve any diffusion the growth of lath or plate is very fast. However bainitic transformation involves diffusion therefore the growth rate is much slower. As against this, pearlitic transformation involves complete reconstruction of the structure. It begins with the nucleation and growth of alternate layer of cementite ferrite at austenite boundary. There is no definite orientation relationship between cementite (cm) or ferrite (α) with austenite (γ). It does result in any surface undulation.

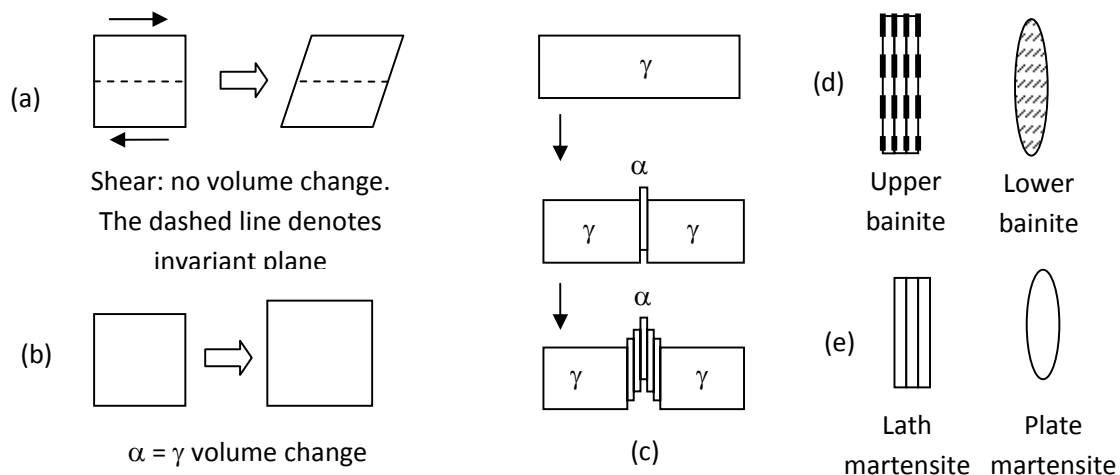


Fig 5: Illustrates how bainitic and martensitic transformation takes place in steel with the nucleation of ferrite lath or plates. This involves shear where there is no change in volume. The plane on which shear occurs remains undistorted. This is denoted by dashed line in (a). It also represents the habit plane. Nucleation of ferrite is also associated with a change in volume. This is illustrated in (b). It has to be accommodated as dilation and strain energy. The sketches in (c) show how it occurs within a super-cooled grain of austenite. If it occurs as a packet of several thin lath or plate the total strain energy is much less. Note that the surface becomes uneven as the plate or lath form within austenite. The sketches in (d) show the structural features of bainite. In upper bainite there are carbides along the lath boundary whereas in lower bainite the carbides are within the plate oriented at an angle with its longer axis. The sketches in (e) show the main feature of lath and plate martensite. However unlike bainite there is no precipitate.

Summary:

In this module we have looked at pearlitic, bainitic and martensitic transformations in eutectoid steel. The first two transformations are diffusion controlled whereas the last one is diffusion-less (it occurs without any diffusion). The slide 8 gives a comparison of the three types of transformations in steel. The diffusion controlled part of the TTT diagram has a characteristic 'C' shape. Both nucleation & growth play a major role in all diffusion controlled transformation. A simple model based approach has been introduced to derive the shape of the fraction transformed versus time plot at a given temperature. It has a characteristic 'S' shape. The equation used to represent this has two parameters, time exponent n and characteristic time τ . The exponent n is an indicator of the mechanism of transformation. Isothermal transformation of austenite below the eutectoid temperature follows Johnson-Mehl-Avrami equation where the time exponent is 4. The expression for the characteristic time τ can be used to explain the origin of 'C' shaped TTT diagrams for such transformations. The effect of %C on the TTT diagram was also discussed. Such plots have an additional curve that represents the time at which precipitation of pro-eutectoid (ferrite or cementite) phase begins. Besides this with the increase in %C the diffusion controlled transformation becomes slow. The 'C' shaped curves shift to the right. The M_s and M_f temperatures decrease with increasing %C.

Pearlite Bainite & Martensite			
	Pearlite	Bainite	Martensite
Structure	Lamellar $\alpha + \text{cm}$	Lath: $\alpha + \text{cm}$ Acicular : $\alpha + \text{Fe}_{2.4}\text{C}$	Acicular, single phase
Diffusion	Long range	Short range	No diffusion
Transformation	Isothermal	Isothermal	Athermal
Orientation relationship	none	Definite relationship	Definite relationship

Slide 8

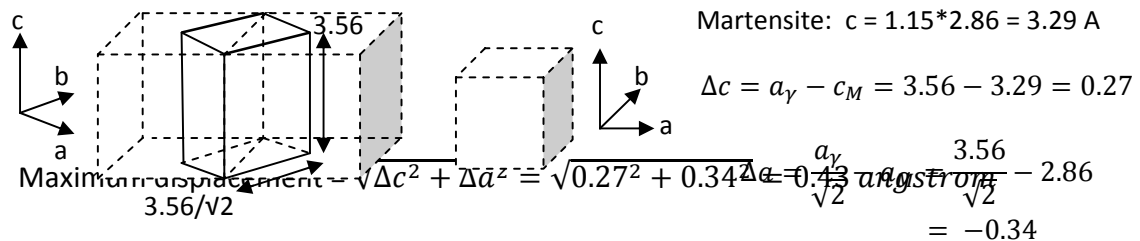
Exercise:

1. Use the extended volume concept to show that if diffusion controlled transformation proceeds at constant nucleation rate (N) and constant growth rate (G) to show that fraction transformed (f) in time t is given by $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$
2. The growth of pearlite nodule at a give temperature in eutectoid steel is known to follow the following relationship: $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$ where f is fraction transformed, t is time, N is nucleation rate and G is growth rate. Use this to explain the characteristic shape of TTT diagram and show that the average nodule size of Pearlite is proportional to: $\left(\frac{G}{N}\right)^{\frac{1}{4}}$.
3. Use the concept of Bain distortion to estimate maximum displacement experienced by iron atom during martensitic transformation. Lattice parameters of austenite and ferrite are 0.356nm & 0.286nm respectively. Assume c/a ratio of martensite to be 1.15.
4. Name the three most important characteristics of martensitic transformation in steel.

Answer:

1. Assume the shape of nuclei to be spherical. If both N & G are constant at any instant t. Number of new nuclei formed in time dt = Ndt. If G is the average growth rate the average radius of preexisting nuclei = Gt. Therefore increase in extended volume due to transformation: $df_{ext} = \frac{4\pi}{3}(Gt)^3Ndt$. Since $df = (1-f)df_{ext} \therefore df = (1-f)\frac{4\pi}{3}NG^3t^3$
On integration: $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$.
2. Nucleation rate (N) depends on the degree of super cooling. Lower the transformation temperature higher is the magnitude of N. Whereas G depends on diffusion of carbon atoms in austenite; it becomes slow if temperature is low. Therefore rate of transformation is the higher at intermediate temperatures. It is reflected in the C shape of TTT diagram. To estimate nodule size assume that transformation is nearly complete say 95%. If it happens in time $t_{0.95}$ $Gt_{0.95}$ is the average nodule radius. Since $f = 1 - \exp\left(-\frac{\pi}{3}NG^3t^4\right)$ or, $\left(\frac{\pi}{3}NG^3t^4\right) = \ln\left(\frac{1}{1-f}\right)$ Average diameter of nodule $D = 2Gt$. When $f = 0.95$ $\ln\left(\frac{1}{1-f}\right) \approx 3$ & $\frac{\pi}{3}NG^3t^4 = \frac{\pi}{48}\left(\frac{N}{G}\right)(2Gt)^4$. Therefore on equating the two: $D = \left[\frac{144}{\pi}\frac{G}{N}\right]^{\frac{1}{4}}$. We get finer nodule if G is small & N is large. Finest Pearlite nodule is expected at the knee of the TTT diagram.

3. The following diagram shows how fcc austenite gets transformed into bct martensite. The cubes drawn in dotted line is austenite lattice. The tetragon drawn with firm line is a bct unit cell with higher c/a ratio. It changes to martensite unit cell by contraction along c axis and expansion along the other two axes.



4. Three most important characteristics of martensitic transformation in steel are (a) it is diffusion less there is no change in composition (b) it is athermal: takes place when the temperature goes below a critical temperature and the transformation continues as long as the temperature decreases (c) it results in a product having a very fine structure and high hardness.