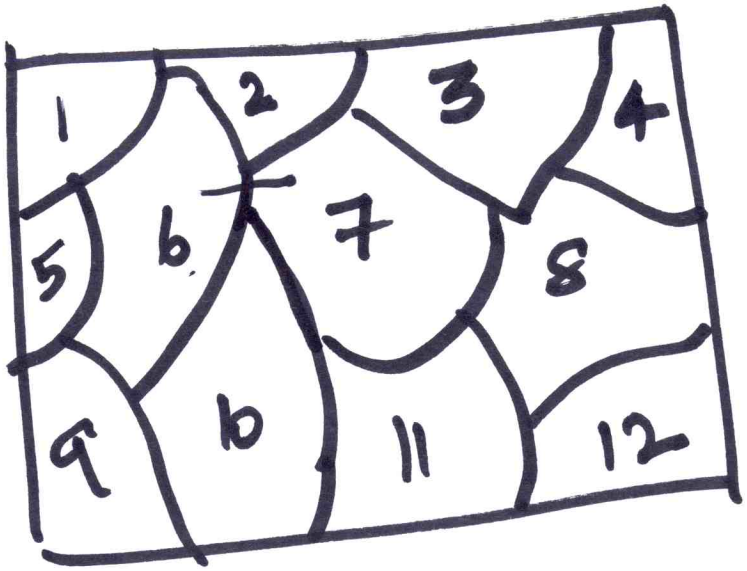
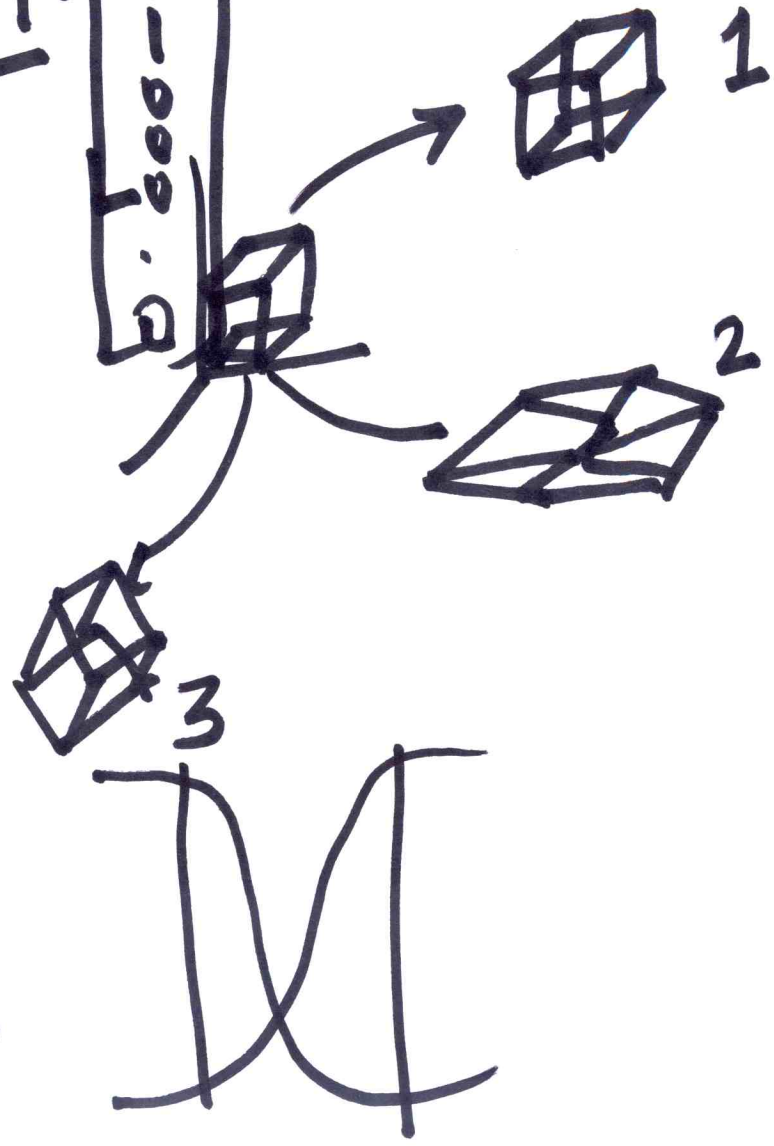
Prof. M. P. Gururajan
Lec. No. 76/77
Date: 19/3/16



$\eta = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$



$\eta_i, \quad i = 1, \ldots 12$

$\eta_6 = 1, \quad \eta_{i \neq 6} = 0$

$\eta_7 = 1, \quad \eta_{i \neq 7} = 0$

We need minima for each η_i

The bulk free energy density f_0 should be the same for all η_i

Fan and Chen – 1996/1997

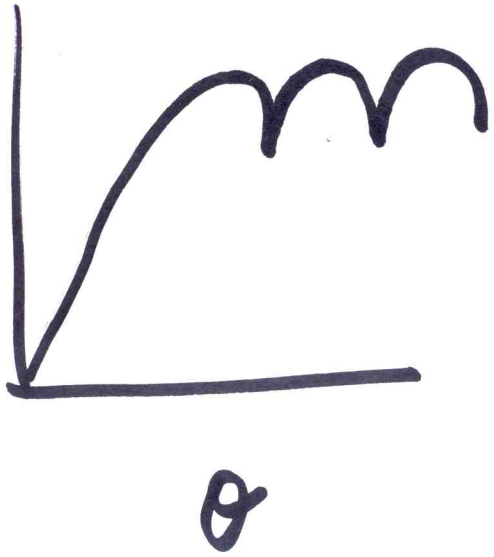
$$f_0(\eta_1, \eta_2 \ldots \eta_p) = \sum_{i=1}^{p} -\frac{\alpha \eta_i^2}{2} + \frac{\beta \eta_i^4}{4}$$

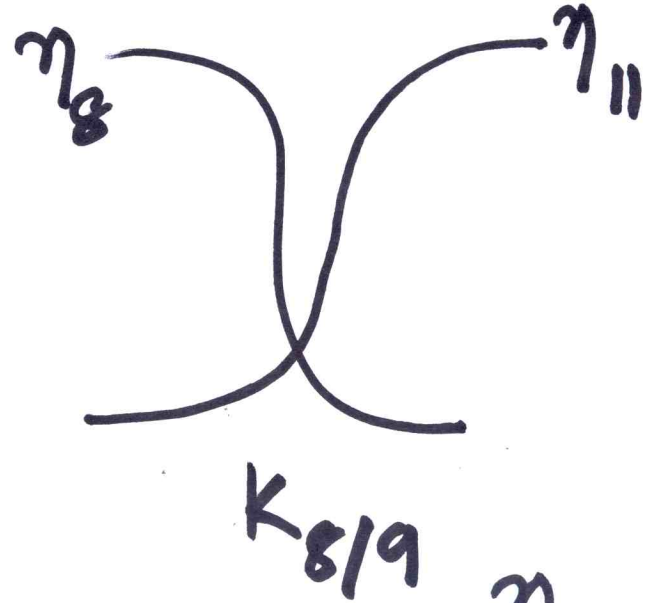
$$+ \gamma \sum_{i=1}^{p} \sum_{j \neq i}^{p} \eta_i^2 \eta_j^2$$

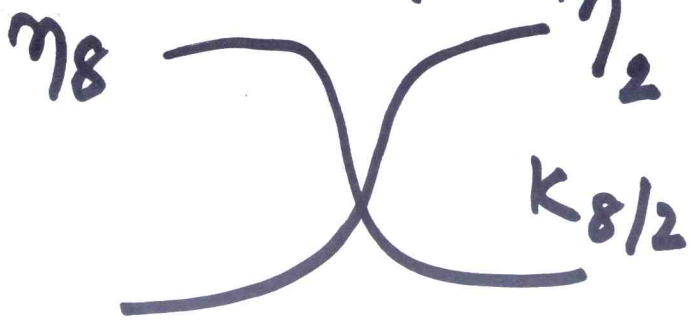
2p minima $\eta_i = \pm k$, $\eta_{j \neq i} = 0$

$$\gamma > \frac{\beta}{2}.$$

$$F = \int \{ f_0(\eta_1, \eta_2 \ldots \eta_p) + \sum_i \underline{\underline{K_i}} |\nabla \eta_i|^2 \} \, dV$$





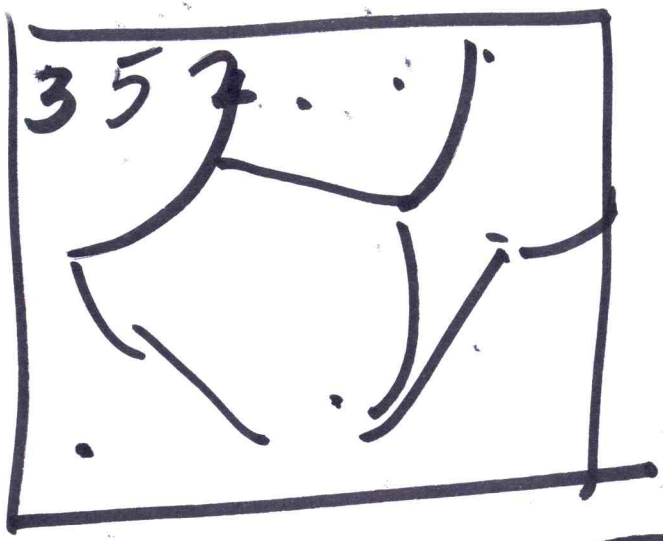


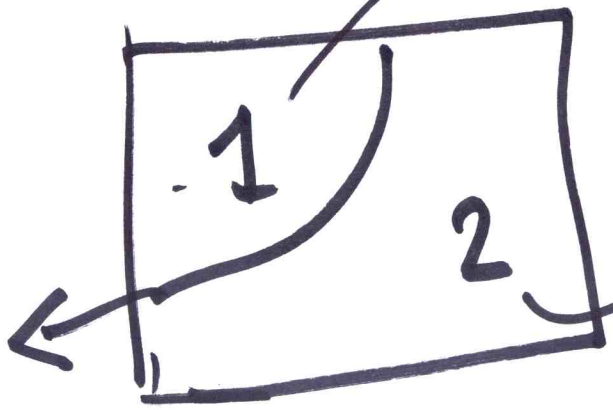
$$\frac{\partial \eta_i}{\partial t} = -L_i \left(\underbrace{\frac{\partial f_0}{\partial \eta_i}}_{g_i} - 2k_i \nabla^2 \eta_i \right) \qquad i = 1, 2, \cdots p$$

$$\frac{\partial \tilde{\eta}_i}{\partial t} = -L_i \, \tilde{g}_i^{\,t} - 2k^2 L_i k_i \, \tilde{\eta}_i^{\,t+\Delta t}$$

$$\frac{\tilde{\eta}_i^{\,t+\Delta t} - \tilde{\eta}_i^{\,t}}{\Delta t} = -L_i \tilde{g}_i^{\,t} - 2 L_i k_i k^2 \tilde{\eta}_i^{\,t+\Delta t}$$

$$\boxed{\tilde{\eta}_i^{\,t+\Delta t} = \frac{\tilde{\eta}_i^{\,t} - L_i \Delta t \, \tilde{g}_i^{\,t}}{1 + 2 L_i k_i k^2 \Delta t}}$$





$\eta_1 = 1 ; \eta_2 = 0$

$\eta_2 = 1 ; \eta_1 = 0$

$\eta_1 \neq 0$
$\eta_2 \neq 0$

$$b = \sum_{i \neq j} \eta_i \eta_j$$

$$f_0 = \sum_{i=1}^{p} -\frac{\alpha \eta_i^2}{2} + \frac{\beta \eta_i^4}{4} + \gamma \sum_{i=1}^{p} \sum_{j \neq i}^{p} \eta_i^2 \eta_j^2$$

$$\frac{\partial f_0}{\partial \eta_i} = g_i = -\alpha \eta_i + \beta \eta_i^3 + 2\gamma \eta_i \sum_{j \neq i}^{p} \eta_j^2$$