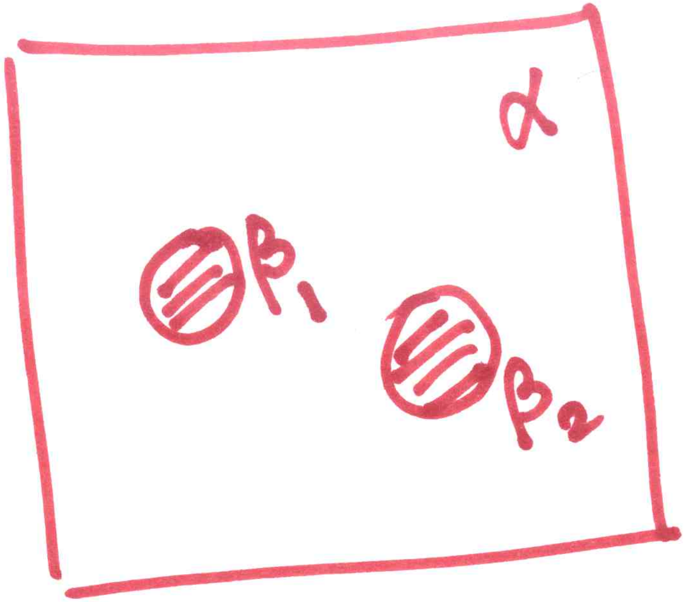
Prof. M.P. Gururajan
Lec. No. 74, 75
Date: 9/3/16

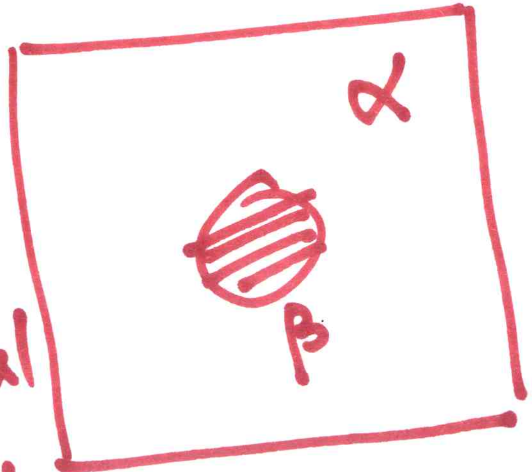


α – A-rich matrix

β_1, β_2 – B-rich.

β_1 – Crystal structure $\neq$ β_2 – Crystal structure

C – η_1
C – η_2



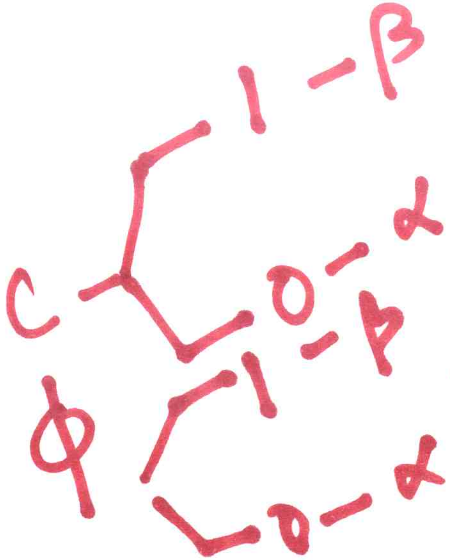
α – β

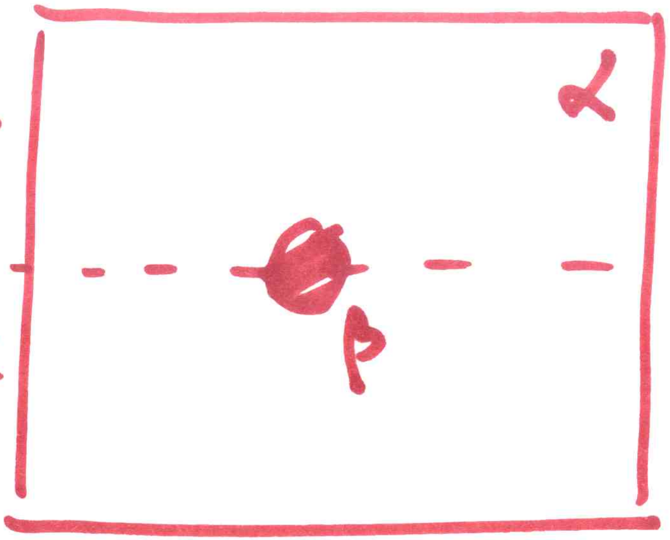
Differ in Composition

Differ in crystal structure

C, ϕ → Crystal Structure

C ↓ α / β

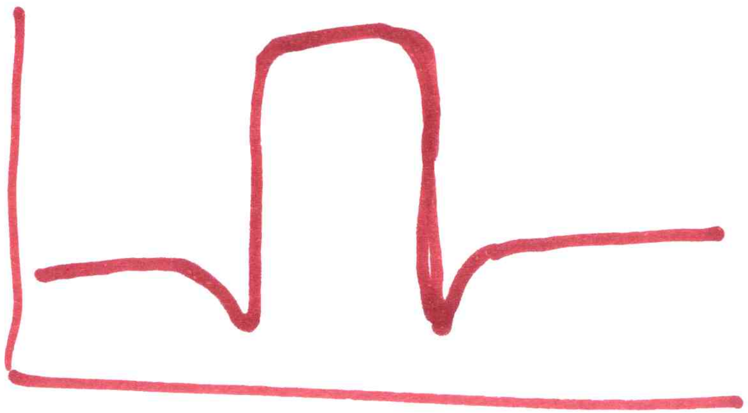




α-Supersaturated

Zener-Frank

Isolated precipitate

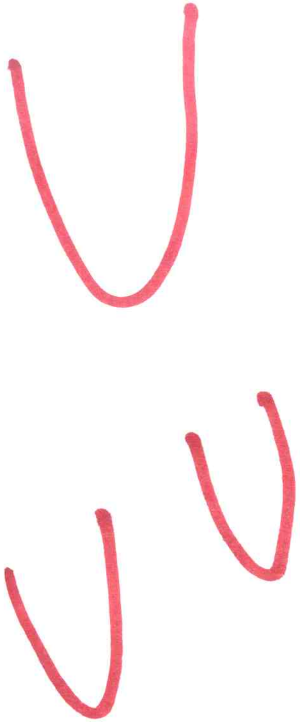


1D
2D
3D

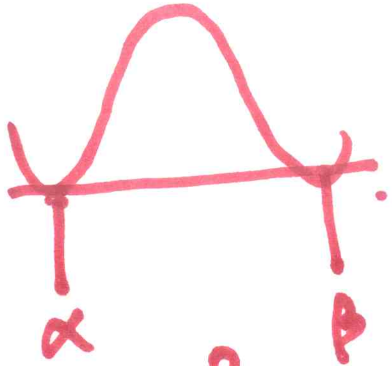
Growth rates

Assume No Gibbs-Thomson effect //

Diffusivity D - Constant



$$M \frac{\partial^2 f}{\partial c^2} \equiv D$$



$$\left.\frac{\partial^2 f}{\partial c^2}\right|_{c=0} = 2A$$

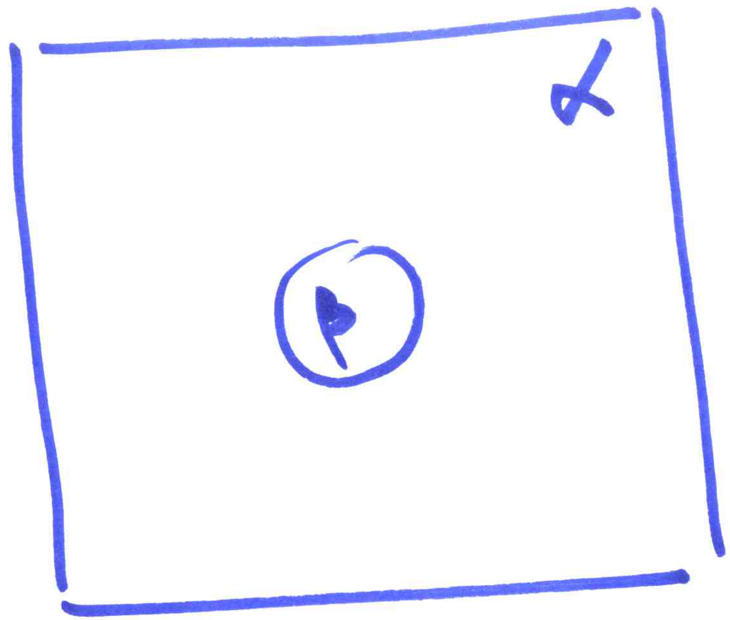
$$f = A c^2 (1-c)^2$$

$$\left.\frac{\partial^2 f}{\partial c^2}\right|_{c=1} = 2A$$

$$\frac{\partial f}{\partial c} = 2Ac(1-c)(1-2c)$$

$$\left.\frac{\partial^2 f}{\partial c^2}\right|_{c=0.5} = -2A$$

$$\frac{\partial^2 f}{\partial c^2} = 2A(1-c)(1-2c) - 2Ac(1-2c) - 4Ac(1-c)$$



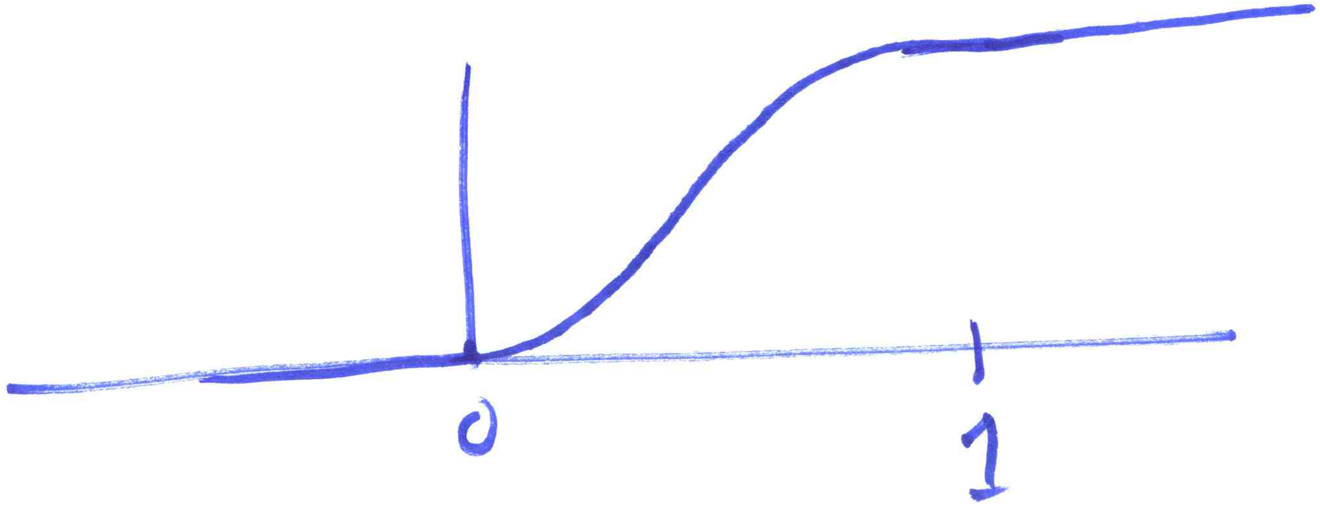
c, ϕ

$$\frac{F}{N_V} = \int \Big[\underbrace{f(c,\phi)} + \kappa_c (\nabla c)^2 + \kappa_\phi (\nabla \phi)^2\Big]\, dV$$

$$\frac{\partial f}{\partial c} - \nabla \frac{\partial (F/N_V)}{\partial \nabla c} \Big/ \frac{\partial f}{\partial \phi} - \nabla \frac{\partial F/N_V}{\partial \nabla \phi}$$

$$f(c,\phi) = Ac^2(1-W(\phi)) + B(1-c)^2 W(\phi)$$
$$+ P\phi^2(1-\phi)^2$$

$$W(\phi) = \begin{cases} 0 & \text{if } \phi < 0 \\ \phi^3(10-15\phi+6\phi^2) & \text{if } 0 \le \phi \le 1 \\ 1 & \text{if } \phi > 0 \end{cases}$$



$$\frac{\mathcal{F}}{N} = \int \left\{ f(c,\phi) + \kappa_c (\nabla c)^2 + \kappa_\phi (\nabla \phi)^2 \right\} dV$$

$$f(c,\phi) = A c^2 (1 - w(\phi)) + B (1-c)^2 \, w(\phi) + P \phi^2 (1-\phi)^2$$

$$w(\phi) = \begin{cases} 0 & \text{if } \phi < 0 \\ \phi^3 (10 - 15\phi + 6\phi^2) & \text{if } 0 \le \phi \le 1 \\ 1 & \text{if } \phi > 1 \end{cases}$$

$$\frac{\partial c}{\partial t} = M \nabla^2 \left(\frac{\delta (\mathcal{F}/N_v)}{\delta c} \right) \qquad \frac{\partial \phi}{\partial t} = -L \frac{\delta (\mathcal{F}/N_v)}{\delta \phi}$$

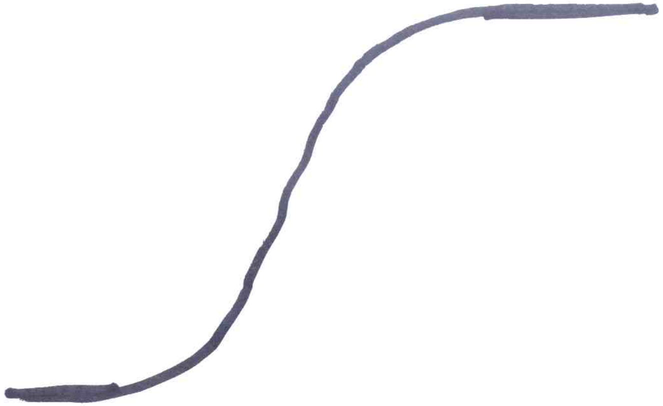
$$\frac{\delta (F/N_v)}{\delta c} = \frac{\partial f(c,\phi)}{\partial c} - 2K_c \nabla^2 c$$

$$\frac{\delta (F/N_v)}{\delta \phi} = \frac{\partial f(c,\phi)}{\partial \phi} - 2K_\phi \nabla^2 \phi$$

$$h_c = \frac{\partial f(c,\phi)}{\partial c} = 2Ac\,(1-W(\phi)) - 2B(1-c)\,W(\phi)$$

$$h_\phi = \frac{\partial f(c,\phi)}{\partial \phi} = -Ac^2 W'(\phi) + B(1-c)^2 W'(\phi) + 2P\phi(1-\phi)(1-2\phi)$$

$$W'(\phi) = \begin{cases} 0 & \text{if } \phi < 0 \\ 3\phi^2(10 - 15\phi + 6\phi^2) \\ \quad + \phi^3(12\phi - 15) \\ 0 & \text{if } \phi > 1 \end{cases}$$



$$\frac{\partial c}{\partial t} = M \nabla^2 \left(h_c(\phi, c) - 2K_c \nabla^2 c \right)$$

$$\frac{\partial \phi}{\partial t} = -L \left(h_\phi(c, \phi) - 2K_\phi \nabla^2 \phi \right)$$