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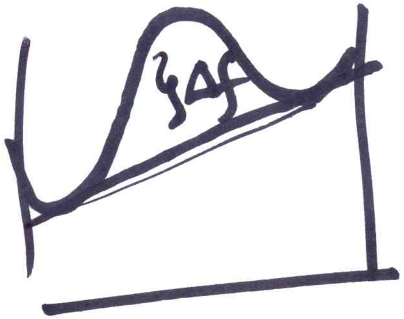
Lec No 65 to 69
Date 26/2/16

$$\frac{F}{N_v} = f_0(c) + \kappa\left(\frac{dc}{dx}\right)^2$$

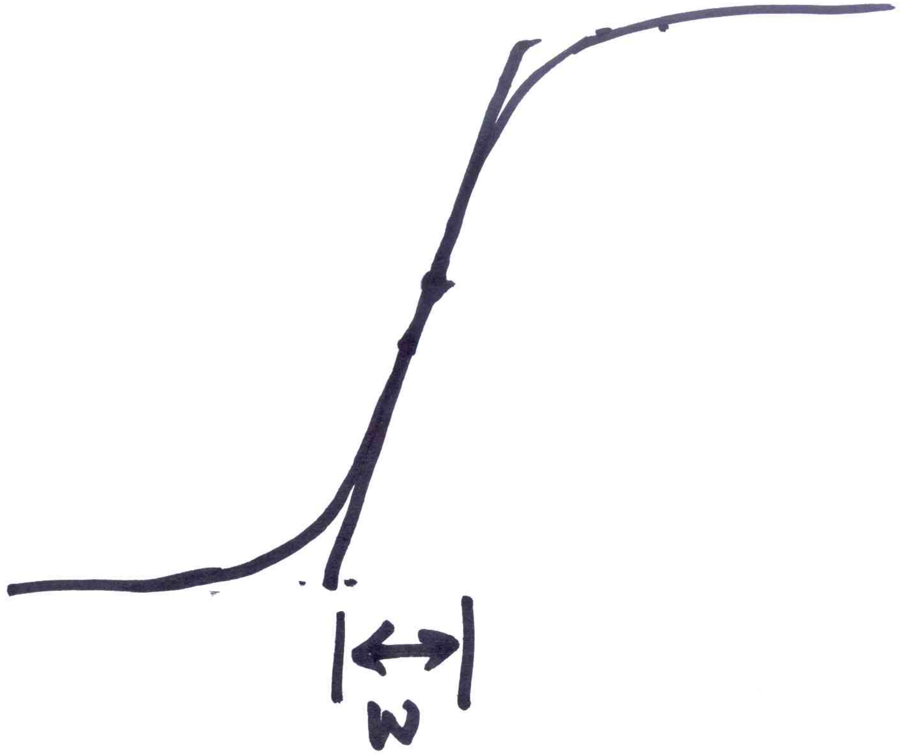


$$\frac{\mathcal{F}}{N_v} = \int F\, dx.$$

$$\sigma = \int \left[F - c_B \mu_B - (1 - c_B)\, \mu_A\right] dx$$

$$\sigma = \int \left[\Delta f + \kappa\left(\frac{dc}{dx}\right)^2\right] dx$$

$$\frac{f_0}{K} = \left(\frac{dc}{dx}\right)^2$$



$$\frac{dc}{dx} = \sqrt{\frac{f_0}{K}}$$

$$w \sim \sqrt{\frac{A}{K}}$$

$$\sigma \sim \sqrt{AK}$$

$$\sigma = 2\gamma \int_0^1 (c - c^2)\, dc$$

$$= 2\gamma \left[\frac{c^2}{2} - \frac{c^3}{3} \right]_0^1$$

$$= 2\gamma \left[\frac{1}{2} - \frac{1}{3} \right]$$

$$\gamma = \sqrt{KA}$$

$$= \gamma \left[1 - \frac{2}{3} \right]$$

$$\boxed{\sigma = \frac{\gamma}{3}}$$

$$K, A = 1$$

$$\sigma = 2\int_0^1 f_0 \left(\frac{k}{f_0}\right)^{1/2} dc$$

$$= 2\int_0^1 (f_0 k)^{1/2} dc$$

$$= 2\int_0^1 \left(Ak\, c^2 (1-c)^2\right)^{1/2} dc$$

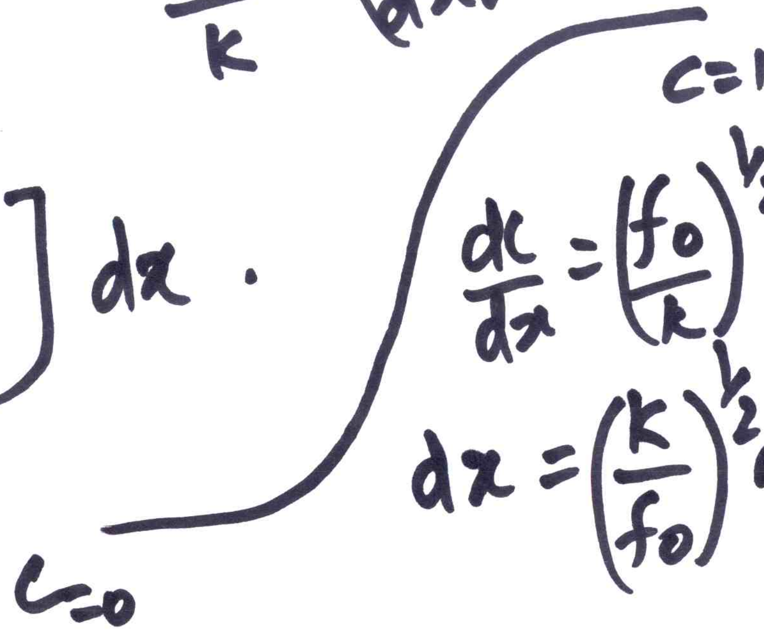
$$\gamma = (Ak)^{1/2}$$

$$= 2\gamma \int_0^1 c(1-c)\, dc = 2\gamma \int_0^1 (c - c^2)\, dc$$

$$K\left(\frac{dc}{dx}\right)^2 = A c^2(1-c)^2$$

$$= f_0$$

$$\frac{f_0}{K} = \left(\frac{dc}{dx}\right)^2$$



$$\frac{dc}{dx} = \left(\frac{f_0}{K}\right)^{1/2}$$

$$dx = \left(\frac{K}{f_0}\right)^{1/2} dc$$

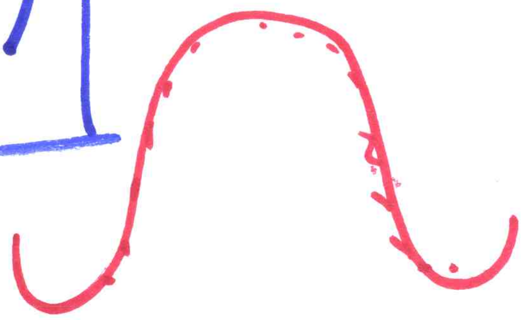
$$\sigma = \int \left[f_0 + K\left(\frac{dc}{dx}\right)^2\right] dx .$$

$$= 2\int_{-\infty}^{\infty} f_0 \, dx$$

$$= 2\int_0^1 \underbrace{A c^2(1-c)^2}_{f_0} \cdot \left(\frac{K}{f_0}\right)^{1/2} dc$$

$$\frac{1}{1+e^{-2x}} = \frac{1}{2}\left(1+\tanh x\right)$$

$$\boxed{c = \frac{1}{1+e^{-\beta x}} = \frac{1}{2}\left(1+\tanh \frac{\beta x}{2}\right)}$$



$$\beta = \sqrt{\frac{A}{K}}$$

$$F = A c^2 (1-c)^2 + K\left(\frac{dc}{dx}\right)^2$$

$$C = \frac{1}{1+e^{-\beta x}} = \frac{e^{\beta x}}{1+e^{\beta x}}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$= \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$\tanh x + 1 = 1 + \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$= \frac{1 + \cancel{e^{-2x}} + 1 \cancel{- e^{-2x}}}{1 + e^{-2x}}$$

$$\frac{1 + \tanh x}{2} = \frac{1}{1 + e^{-2x}}$$

$$\frac{dc}{c} - \frac{(-dc)}{1-c} = \beta\, dx.$$

$$\ln c - \ln(1-c) = \beta x$$

$$\ln \frac{c}{1-c} = \beta x$$

$$\frac{1-c}{c} = e^{-\beta x}$$

$$\frac{1}{c} - 1 = e^{-\beta x}. \Rightarrow \frac{1}{c} = 1 + e^{-\beta x}$$

$$\boxed{c = \frac{1}{1+e^{-\beta x}} = \frac{e^{\beta x}}{1+e^{\beta x}}}$$

$$\frac{A}{c} + \frac{B}{1-c} = \frac{1}{c(1-c)} \qquad \frac{1}{c} + \frac{1}{1-c} = \frac{1-c+c}{c(1-c)}$$

$$= \frac{1}{c(1-c)}$$

$$\frac{A - Ac + Bc}{c(1-c)} = \frac{1}{c(1-c)}$$

$A = 1$; $\quad B - A = 0$. $\Rightarrow B = 1$.

$$\frac{dc}{c} + \frac{dc}{1-c} = \beta\, dx \qquad \beta = \sqrt{\frac{A}{k}}$$

$$\frac{dc}{c} - \frac{(-dc)}{1-c} = \beta\, dx$$

$$K\left(\frac{dc}{dx}\right)^2 - A c^2 (1-c)^2 = 0$$

$$\left(\frac{dc}{dx}\right)^2 = \frac{A}{K} c^2 (1-c)^2 \qquad \beta = \sqrt{\frac{A}{K}}$$

$$\frac{dc}{dx} = \beta c(1-c)$$

$$\frac{dc}{c(1-c)} = \beta \, dx$$

$$\frac{1}{c(1-c)} = \frac{A}{c} + \frac{B}{1-c}$$

$$y' \frac{\partial F}{\partial y'} - F = 0$$

$$\frac{dc}{dx} \cdot \frac{\partial F}{\partial \left(\frac{dc}{dx}\right)} - F = 0$$

$$F = A c^2 (1-c)^2 + K \left(\frac{dc}{dx}\right)^2$$

$$\frac{\partial F}{\partial \left(\frac{dc}{dx}\right)} = 2K \frac{dc}{dx}$$

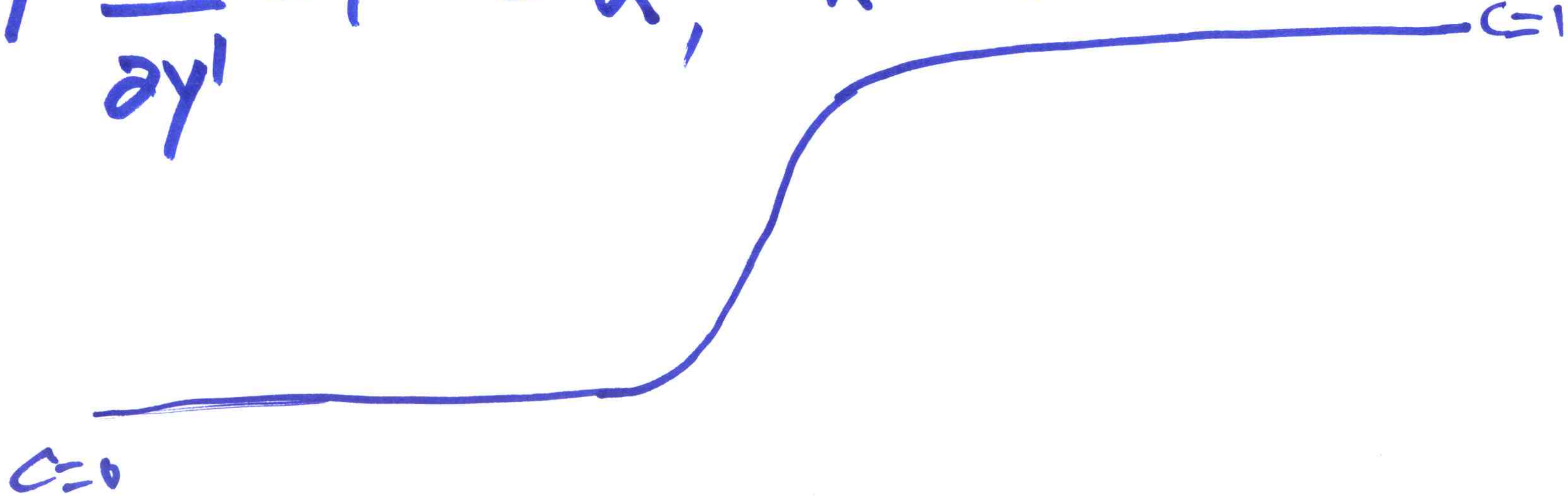
$$2K \left(\frac{dc}{dx}\right)^2 - A c^2 (1-c)^2 - K \left(\frac{dc}{dx}\right)^2 = 0$$

$$\boxed{K \left(\frac{dc}{dx}\right)^2 - A c^2 (1-c)^2 = 0}$$

$$\frac{d}{dx}\left(y'\frac{\partial F}{\partial y'}\right) = \frac{dF}{dx}$$ only if F is not an explicit function of x

$$\frac{d}{dx}\left(y'\frac{\partial F}{\partial y'} - F\right) = 0$$

$$y'\frac{\partial F}{\partial y'} - F = \alpha, \quad \alpha = \text{constant.}$$



$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

$$\frac{\partial F}{\partial y} = \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)$$

$$y'\frac{\partial F}{\partial y} - y'\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

$$\frac{d}{dx}\left(y'\frac{\partial F}{\partial y'}\right) = y'\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) + y''\frac{\partial F}{\partial y'}$$

$$= y'\frac{\partial F}{\partial y} + y''\frac{\partial F}{\partial y'}$$

$$\frac{dF}{dx} = \frac{\partial F}{\partial x} + y'\frac{\partial F}{\partial y} + y''\frac{\partial F}{\partial y'}$$

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

if $F(x, y, y' \equiv \frac{dy}{dx})$

$\downarrow$

$x, c, \quad y' = \frac{dy}{dx}$

$$\frac{\partial F}{\partial c} - \frac{d}{dx}\left(\frac{\partial F}{\partial c'}\right) = 0$$

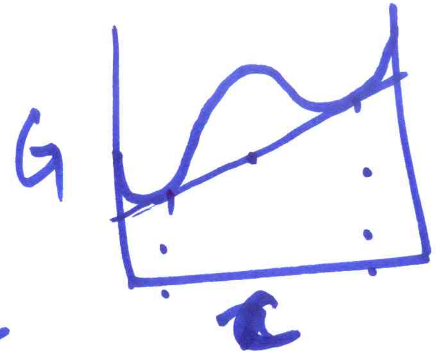
$\mathcal{F} = \int F\,dx$

$$F = f_0(c) + \dot{k}\left(\frac{dc}{dx}\right)^2$$

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

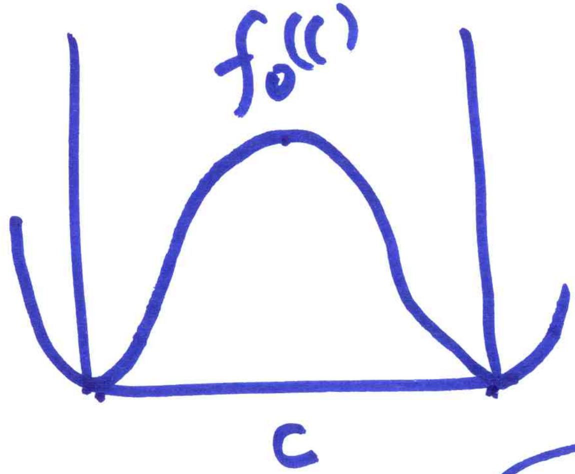
$$y'\frac{\partial F}{\partial y} - y'\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

$$\frac{F}{N_V} = \int \left\{ f_0(c) + K|\nabla c|^2 \right\} dV$$



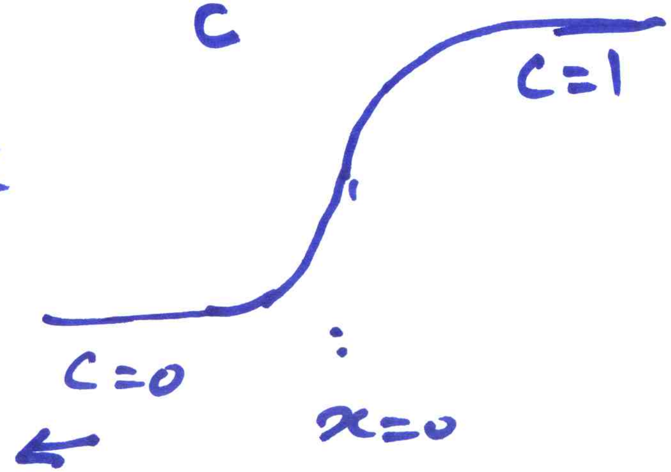
$$\frac{F}{N_V} = \int \left\{ f_0(c) + K\left(\frac{dc}{dx}\right)^2 \right\} dx$$

$$f_0(c) = A c^2 (1-c)^2$$



$$\mu_B c_B^e + (1-c_B^e)\mu_A$$

$$\sigma = \int \underbrace{\left\{ f_0(c) + K\left(\frac{dc}{dx}\right)^2 \right\}}_{F} dx$$



$$C = \frac{1}{1+e^{-\beta x}}$$

$$\beta = \sqrt{\frac{A}{K}}$$

$$\omega \sim \sqrt{\frac{A}{K}}$$

$$\sigma \sim \sqrt{AK}$$

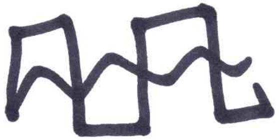
$$\frac{\partial c}{\partial t} = \alpha \nabla^2 c$$

Classical Diffusion

$$\frac{\partial c}{\partial t} = \alpha \nabla^2 c - \beta \nabla^4 c$$

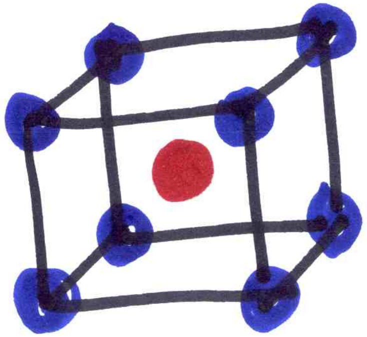
C.H.

Spinodal.



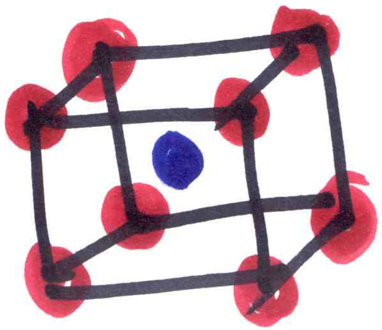
$\Omega > 0$.

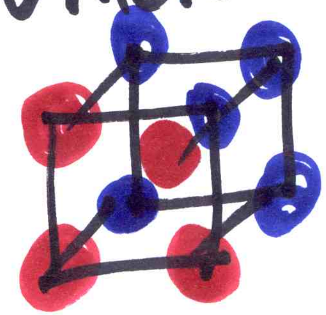
$E_{AB} - \frac{1}{2}(E_{AA} + E_{BB})$



NiAl. B2

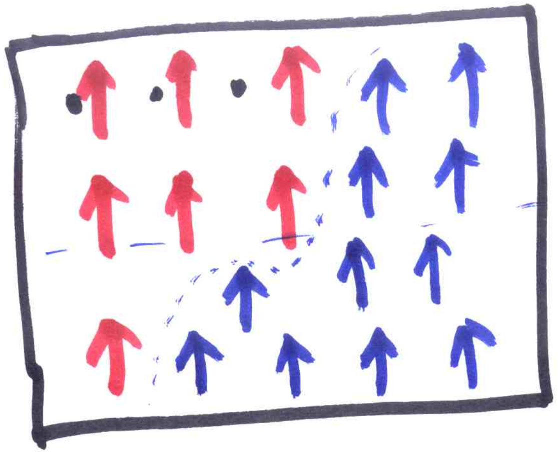
Ordered BCC

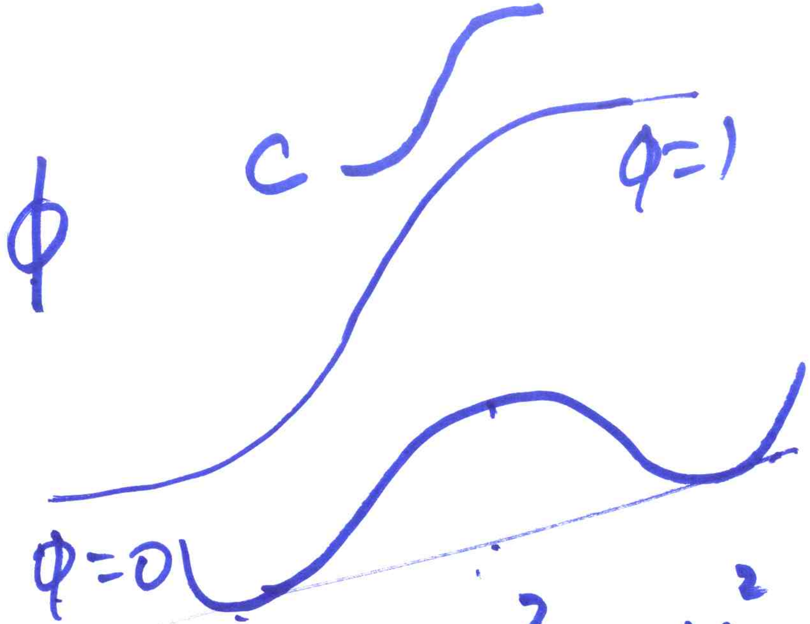




$\Omega < 0$

$E_{AB} - \frac{1}{2}(E_{AA} + E_{BB})$





$$\frac{d}{dt}\int c\,dx = 0$$

$$f_0(\phi) = A\phi^2(1-\phi)^2$$

$$\kappa|\nabla\phi|^2$$

$$\mathcal{F} = \int F\,dV$$

$$= \int \left\{ f_0(\phi) + \kappa|\nabla\phi|^2 \right\} dV$$

$$\frac{\delta \mathcal{F}}{\partial \phi} = \mu .$$

$$\boxed{\frac{\partial \phi}{\partial t} = -L\mu}$$ Allen-Cahn equation

$$\frac{\partial \phi}{\partial t} = -L\left(\frac{\partial f_0}{\partial \phi} - 2K\nabla^2\phi\right)$$

$$\frac{\partial \phi}{\partial t} = \underbrace{2KL}\nabla^2\phi - L\underbrace{\frac{\partial f_0}{\partial \phi}}_{A\phi(1-\phi)(1-2\phi)}$$ Reaction-Diffusion equation

$$\frac{\partial\phi}{\partial t} = 2KL\nabla^2\phi - L\frac{\partial f_0}{\partial\phi}.$$

$$f_0 = A\phi^2(1-\phi)^2.$$

$$\frac{\partial f_0}{\partial\phi} = 2A\phi(1-\phi)(1-2\phi)$$

$$= g(\phi).$$

$$\frac{\tilde{\phi}^{t+\Delta t} - \tilde{\phi}^{t}}{\Delta t} = -2KLk^2\tilde{\phi}^{t+\Delta t} - L\tilde{g}^{t}$$

$$\boxed{\tilde{\phi}^{t+\Delta t} = \frac{\tilde{\phi}^{t} - L\Delta t\,\tilde{g}}{1+2KLk^2}}$$