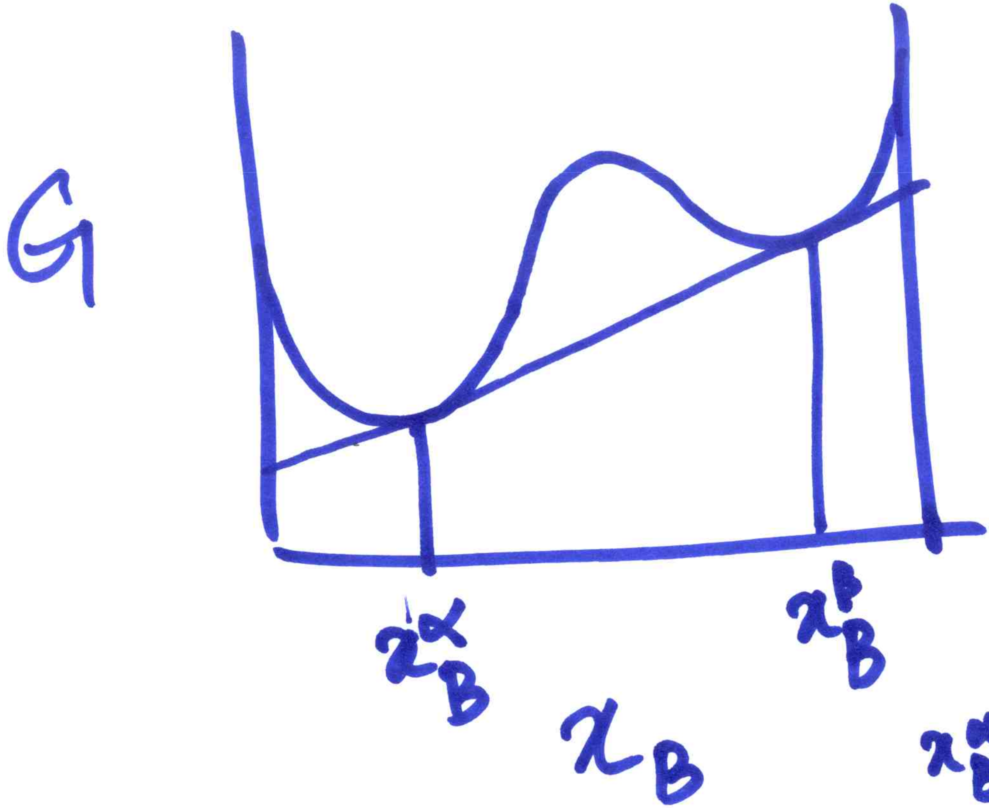
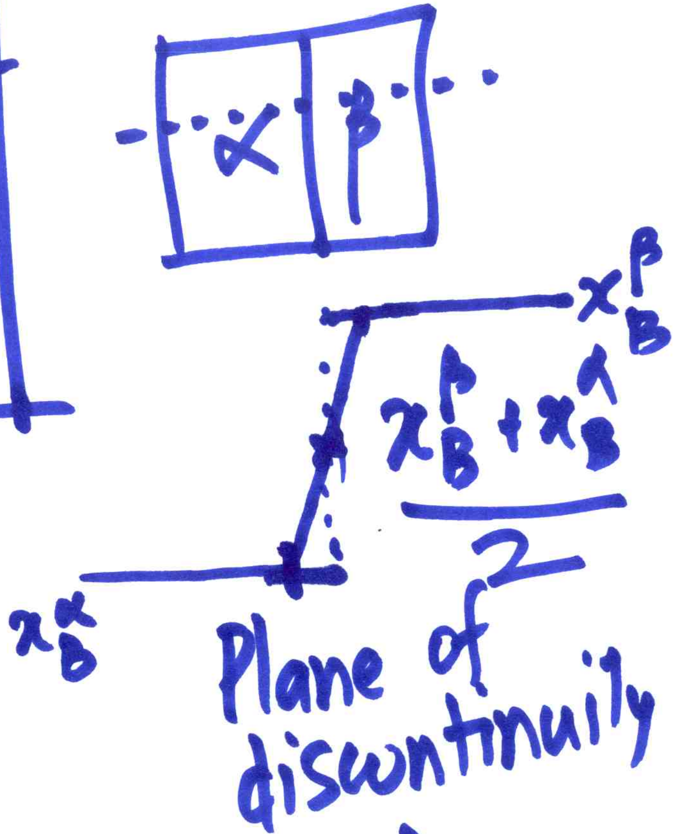




$$\Delta G = \Omega x(1-x) + RT\left(x \ln x + (1-x)\ln(1-x)\right)$$

$$F\left(x, y, z, c, \frac{\partial c}{\partial x}, \frac{\partial c}{\partial y}, \frac{\partial c}{\partial z}, \frac{\partial^2 c}{\partial x^2}, \frac{\partial^2 c}{\partial x \partial y}, \frac{\partial^2 c}{\partial x \partial z}, \dots\right)$$

$$= \int f \, dV$$

$$f = f(c) + \frac{\partial f}{\underbrace{\partial \nabla c}_{\alpha_i}} \cdot \underbrace{\nabla c}_{(\nabla c)_i \;\; \frac{\partial c}{\partial x_i}} + \overbrace{\frac{\partial f}{\partial (\nabla^2 c)_{ij}}}^{\beta_{ij}} : \underbrace{(\nabla^2 c)_{ij}}_{\frac{\partial^2 c}{\partial x_i \partial x_j}} + \frac{1}{2!} \underbrace{\frac{\partial^2 f}{\partial (\nabla c)_i \, \partial (\nabla c)_j}}_{\gamma_{ij}} : (\nabla c)_i (\nabla c)_j + \dots$$

$$f = f(c) + \alpha_i (\nabla c)_i + \beta_{ij} (\nabla^2 c)_{ij} + \frac{\gamma_{ij}}{2} (\nabla c)_i (\nabla c)_j + \cdots$$

$$F = \int f \, dV$$ Free energy functional.

$$\alpha_i \equiv 0$$ (Inversion symmetry)

$$\int \beta_{ij} (\nabla^2 c)_{ij} \, dV = \int \beta_{ij} \frac{\partial^2 c}{\partial x_i \partial x_j} \, dV$$

$$= \int \beta_{ij} \frac{\partial}{\partial x_i} \left(\frac{\partial c}{\partial x_j} \right)$$

$$\int \beta_{ij} \frac{\partial}{\partial x_i} . (\nabla c)_j \, dv = \cancel{\Big[\beta_{ij} (\nabla c)_j\Big]_S} \equiv 0$$

$$- \int \frac{\partial \beta_{ij}}{\partial x_i} . (\nabla c)_j . dV$$

$$= - \int \frac{\partial}{\partial c} \beta_{ij} . \frac{\partial c}{\partial x_i} . (\nabla c)_j \, dv$$

$$F = \int \Big\{ f(c) + \underbrace{\Big[\frac{\gamma_{ij}}{2} - \frac{\partial \beta_{ij}}{\partial c} \Big]}_{K_{ij}} (\nabla c)_i (\nabla c)_j \Big\} \, dv$$

$$F = \int \left[f(c) + K_{ij} (\nabla c)_i (\nabla c)_j \right] dV$$

Cubic.

$$\underbrace{K_{ij} = K_{ji}}$$

$$= \int f(c) + K \delta_{ij} (\nabla c)_i (\nabla c)_j \; dV$$

$$(\nabla c)_i (\nabla c)_i$$

$$F = \int \left[f(c) + K (\nabla c)^2 \right] dV$$

$$\frac{1}{N}\frac{\delta F}{\delta c} = \mu \qquad \frac{1}{N}\frac{\partial G}{\partial c} = \mu$$

$$F = \int \left[f(c) + k(\nabla c)^2 \right] dV.$$

$$\frac{\delta F}{\delta c} = \frac{\partial f}{\partial c} - \nabla \cdot 2k(\nabla c)$$

$$\frac{\delta F}{\delta c} = \frac{\partial f}{\partial c} - 2k\nabla^2 c$$

$$\boxed{N\mu = \frac{\partial f}{\partial c} - 2k\nabla^2 c.}$$

$$J = -M \nabla \mu$$

$$\frac{\partial c}{\partial t} = -\nabla \cdot J$$

$$\frac{\partial c}{\partial t} = \nabla \cdot M \nabla \mu .$$

$$= M \nabla^2 \mu .$$

$$\frac{1}{N} \cdot \frac{\partial c}{\partial t} = M \cdot \nabla^2 \left(\frac{\partial f}{\partial c} - 2K \nabla^2 c \right)$$

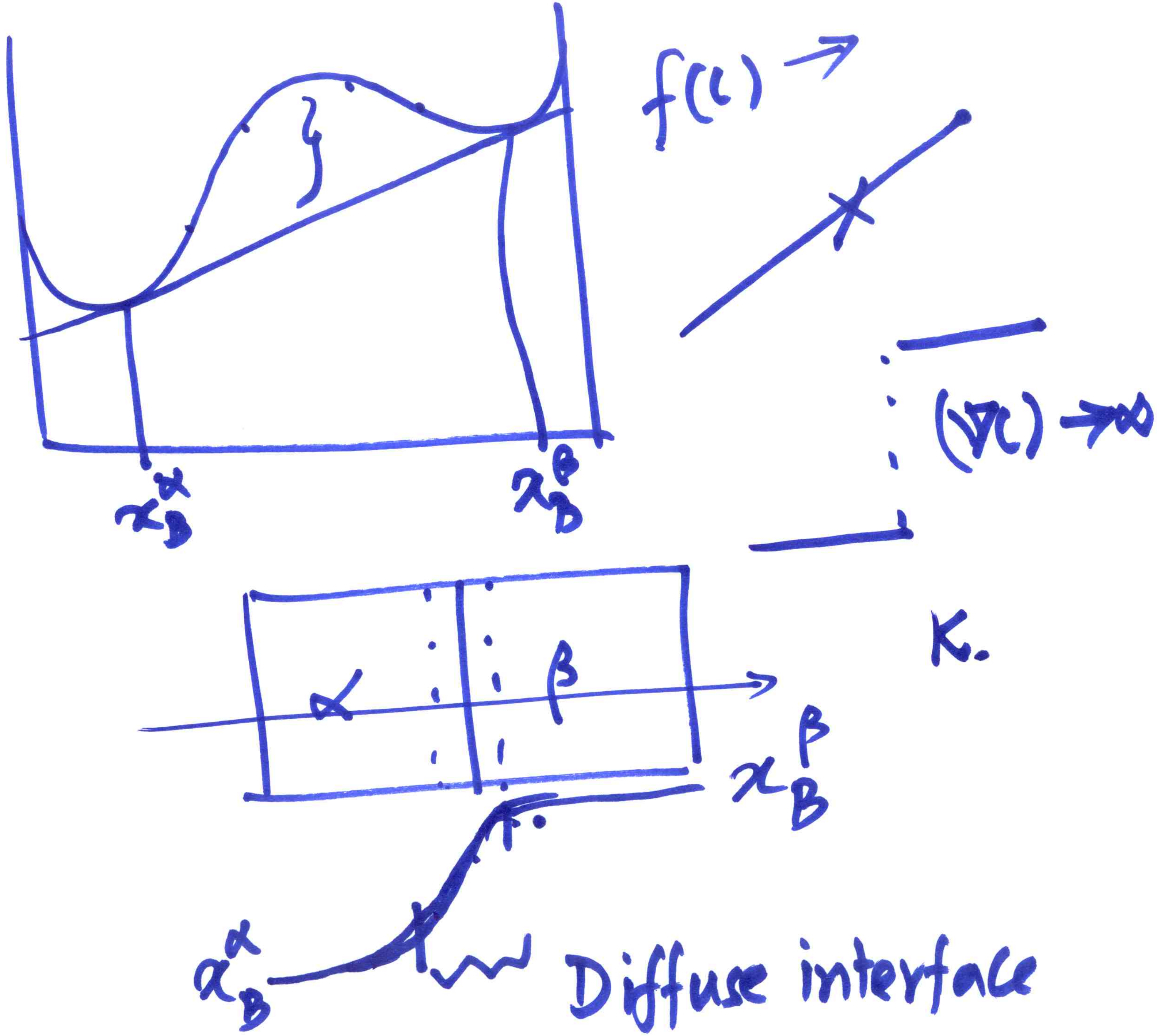
$$\frac{1}{N} \frac{\partial c}{\partial t} = M \nabla^2 \left(\frac{\partial f}{\partial c} \right) - 2KM \nabla^4 c$$

$$\boxed{\frac{\partial c}{\partial t} = M\,\nabla^2\left(\frac{\partial f}{\partial c}\right) - 2kM\nabla^4 c.}$$

Cahn - Hilliard equation

$$\nabla \cdot \frac{\partial f}{\partial c} \Rightarrow \underbrace{\frac{\partial}{\partial x}}_{\frac{\partial}{\partial c}(\;)\cdot\frac{\partial c}{\partial x}} \cdot \frac{\partial f}{\partial c} = \underbrace{\frac{\partial^2 f}{\partial c^2}}_{f''} \cdot \frac{\partial c}{\partial x}.$$

$$\boxed{\frac{\partial c}{\partial t} = M \underbrace{f''}_{D.} \nabla^2 c - 2k M \nabla^4 c}$$



x_B^α
x_B^β
$f(c) \rightarrow$
$(\nabla c) \rightarrow \infty$
K.
α
β
x_B^β
x_B^α
Diffuse interface