$$I = \int_{x_1}^{x_2} F\left(x, y, y' \equiv \frac{dy}{dx}\right) dx$$

Euler-Lagrange

$$\frac{\partial F}{\partial x} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

$$\mathcal{L} = \int_{t_1}^{t_2} (T - V)\, dt = \int \mathcal{F}\left(t, x, \dot{x} \equiv \frac{dx}{dt}\right)$$

$T \rightarrow$ K.E $\frac{1}{2} m \dot{x}^2$

$V \rightarrow$ PE $V(x)$

$$\mathcal{L} = \int_{t_1}^{t_2} \mathcal{F}\left(t, x, \dot{x} \equiv \frac{dx}{dt}\right) dt$$

$$\mathcal{F} = \frac{1}{2} m \dot{x}^2 - V(x)$$

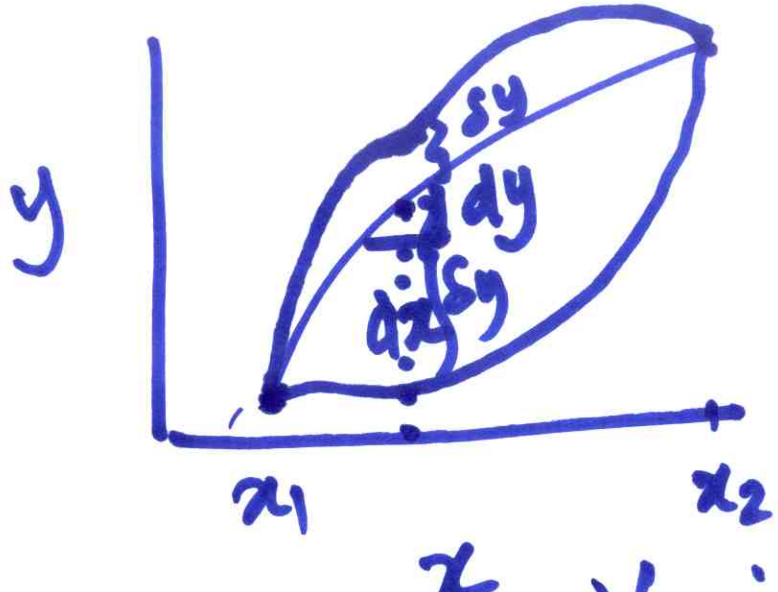
$$\frac{\partial \mathcal{F}}{\partial x} - \frac{d}{dt}\left(\frac{\partial \mathcal{F}}{\partial \dot{x}}\right) = 0$$

$$-\frac{\partial V(x)}{\partial x} - \frac{d}{dt}(m\dot{x}) = 0$$

$$-\frac{\partial V(x)}{\partial x} - m\ddot{x} = 0$$

$$\boxed{F = m\ddot{x}}$$

$$\min \mathcal{L} = \int_{t_1}^{t_2} \mathcal{F}(t, x, \dot{x})\, dt$$



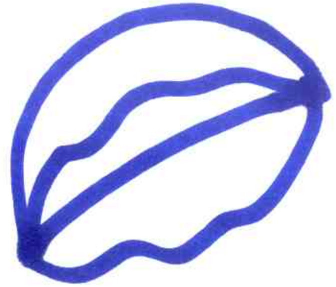
$\frac{dy}{dx}$ - differential

Variation - No change in x

$$\delta I = \tilde{I} - I$$

$$\left.\begin{aligned}\delta\, dy &= d\,\delta y \\ \delta \int y\,dx &= \int \delta y\,dx\end{aligned}\right\}$$

$$I = \int F(x, y, y')\,dx$$



$$\delta y = \tilde{y} - y$$

$$\delta y' = \tilde{y}' - y'$$

$$F(x, y+\delta y, y'+\delta y')$$
$$= F(x, y, y') + \frac{\partial F}{\partial y}\delta y + \frac{\partial F}{\partial y'}\delta y' + O(\delta^2)$$

$$F(x, y+\delta y, y'+\delta y') - F(x, y, y') = \delta^T F$$

$$\delta^T F = \frac{\partial F}{\partial y}\delta y + \frac{\partial F}{\partial y'}\delta y' + O(\delta^2)$$

$$\delta^T I = \int_{x_1}^{x_2} F(x, y+\delta y, y'+\delta y')\,dx.$$

$$= \delta^1(I) + O(\delta^2)$$

$$\delta^1 I = \int_{x_1}^{x_2} \delta^1(F) + O(\delta^2)$$

$$\delta^1 I = \int_{x_1}^{x_2} \left(\frac{\partial F}{\partial y}\delta y + \frac{\partial F}{\partial y'}\delta y'\right) dx$$

$$\delta^1 I = 0$$

$$\int_{x_1}^{x_2} \left(\frac{\partial F}{\partial y} \delta y + \frac{\partial F}{\partial y'} \delta y' \right) dx$$

$$= \int_{x_1}^{x_2} \left[\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'} \right) \right] \delta y \, dx = 0$$

$$+ \cancel{\left[\frac{\partial F}{\partial y'} \delta y \right]_{x_1}^{x_2}}$$

$$\boxed{\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0}$$

E-L. equation

$$\frac{\partial F}{\partial y'}\,\delta y\Big]_{x_1}^{x_2}.$$

Shames and Dym

Energy and FE methods

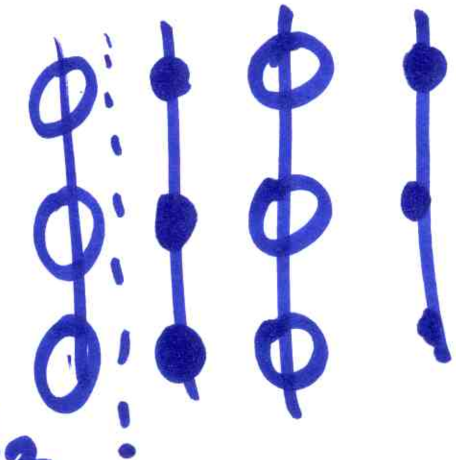
Position/Displacement

$$\frac{\partial \mathcal{F}}{\partial \dot{x}}\,\delta x\Big]_{t_1}^{t_2}$$

Force.

$$G = \Omega x(1-x) + RT\{x\ln x + (1-x)\ln(1-x)\}$$

$$\frac{\partial G}{\partial x} = 0$$



$\Omega < 0$ ordering

$\boxed{\Omega > 0}$ Spinodal

$$G = \int_{x_1}^{x_2} F\left(x, y, z, c, \frac{\partial c}{\partial x}, \frac{\partial c}{\partial y}, \frac{\partial c}{\partial z}, \frac{\partial^2 c}{\partial x^2}, \frac{\partial^2 c}{\partial y^2}, \frac{\partial^2 c}{\partial z^2}, \frac{\partial^2 c}{\partial x \partial y}, \frac{\partial^2 c}{\partial y \partial z}, \frac{\partial^2 c}{\partial z \partial x}, \ldots\right) dy$$

$$G = \int F\left(x, y, z, c, \nabla c, \nabla^2 c, \ldots\right) dV$$

$$x_i, \quad c, \quad \underset{\text{vector}}{\nabla c}$$

$$\begin{pmatrix} \frac{\partial c}{\partial x} \\ \frac{\partial c}{\partial y} \\ \frac{\partial c}{\partial z} \end{pmatrix} \qquad \frac{\partial c}{\partial x_i}$$

$$\nabla^2 c = \underbrace{\frac{\partial^2 c}{\partial x_i \partial x_j}}_{\text{II rank tensor}}$$

$$F(c, \nabla c, \nabla^2 c, \ldots) = f(c) + \frac{\partial f}{\partial(\nabla c)} \cdot \nabla c + \frac{\partial f}{\partial \nabla^2 c} : \nabla^2 c + \frac{\partial^2 f}{\partial \nabla c \, \partial \nabla c} : (\nabla c)^2 + \ldots$$

$$\frac{\partial f}{\partial \nabla c} = \frac{\partial f}{\partial \left(\frac{\partial c}{\partial x_i}\right)} = \alpha_i \left(\cdot \frac{\partial c}{\partial x_i} \equiv \text{Scalar} \right.$$

$$\frac{\partial^2 f}{\partial \nabla^2 c} = \frac{\partial f}{\partial \left(\frac{\partial^2 c}{\partial x_i \partial x_j}\right)} = \beta_{ij} \left(: \frac{\partial^2 c}{\partial x_i \partial x_j} \equiv \text{Scalar} \right.$$

$$\frac{\partial^2 f}{\partial \nabla c \, \partial \nabla c} = \frac{\partial^2 f}{\partial \left(\frac{\partial c}{\partial x_i}\right) \cdot \partial \frac{\partial c}{\partial x_j}} = \gamma_{ij} \cdot \left(: \frac{\partial^2 c}{\partial (\nabla c)^2} \equiv \text{Scalar?} \right)$$