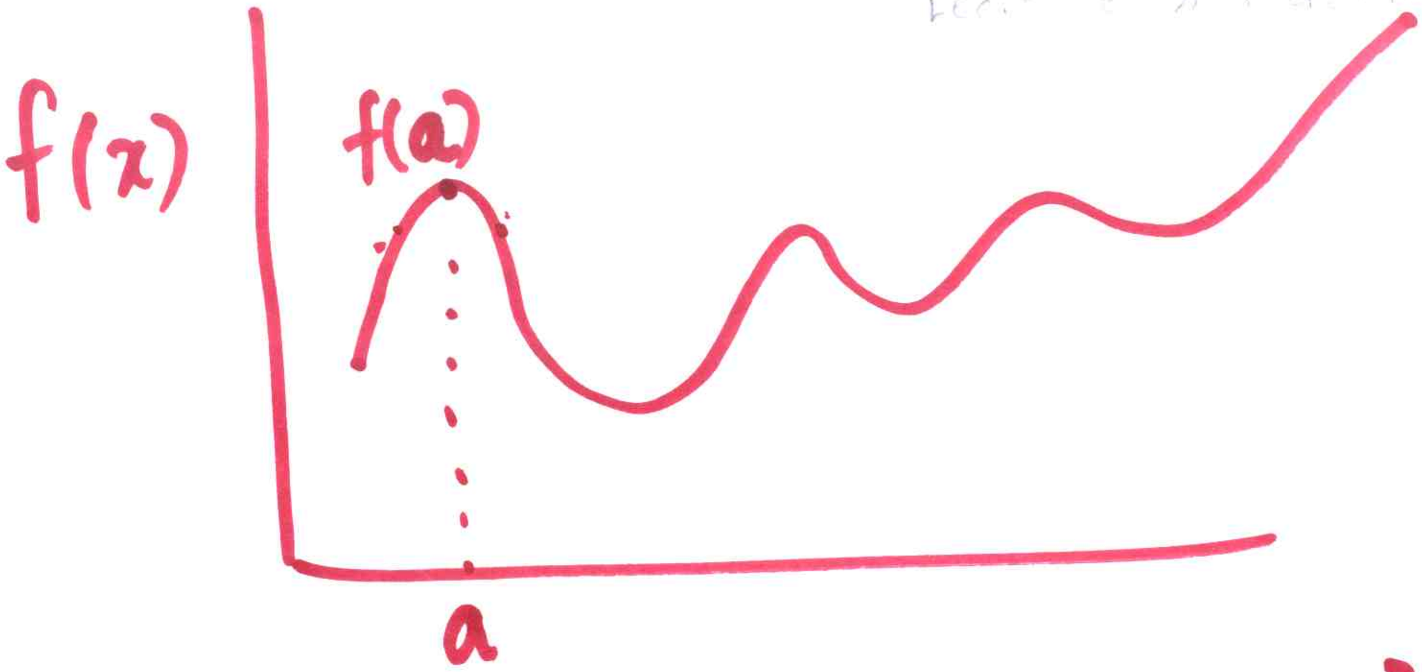
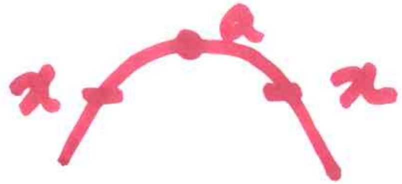


$$f(x) = f(a) + (x-a)\, f'(x)\Big|_{x=a} + \frac{(x-a)^2}{2!} f''(x)\Big|_{x=a} + \cdots$$

$$f' = \frac{df}{dx}$$

$$f(x) - f(a) = (x-a)\, f'(x)\Big|_{x=a} + \frac{(x-a)^2}{2!} f''(x)\Big|_{x=a} + \cdots$$

$$f(x) - f(a) < 0$$



$$(x-a) f'(x)\Big|_{x=a} + \frac{(x-a)^2}{2!} f''(x)\Big|_{x=a} + \cdots$$

$$f'(x)\Big|_{x=a} = 0$$

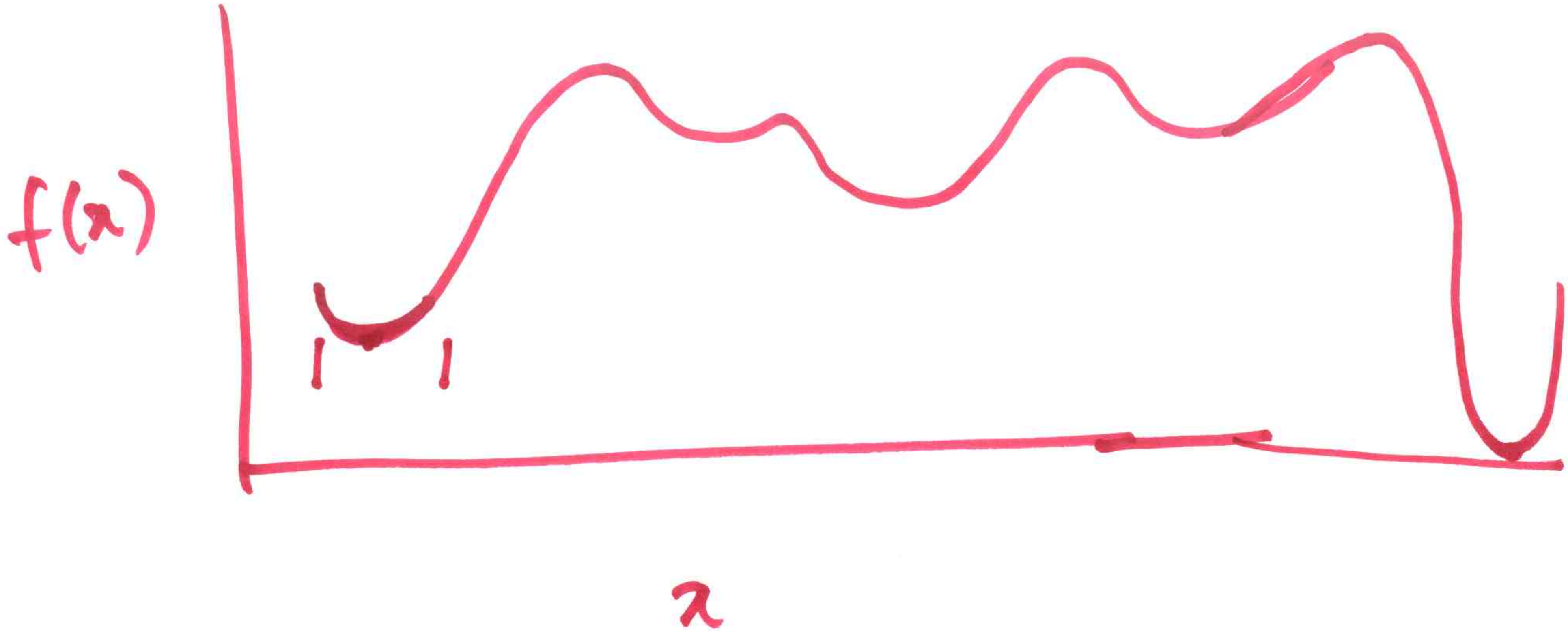
$$f''(x)\Big|_{x=a} < 0$$

$$f(x) - f(a) > 0$$

$$(x-a)\, f'(x)\Big|_{x=a} + \frac{(x-a)^2}{2!} f''(x)\Big|_{x=a} + \ldots$$

$$f'(x)\Big|_{x=a} = 0$$

$$f''(x)\Big|_{x=a} > 0$$



Admissible x

$f'(x), f''(x) \dots$ exist.

$$\mathcal{L} = T - V$$

T: Kinetic energy

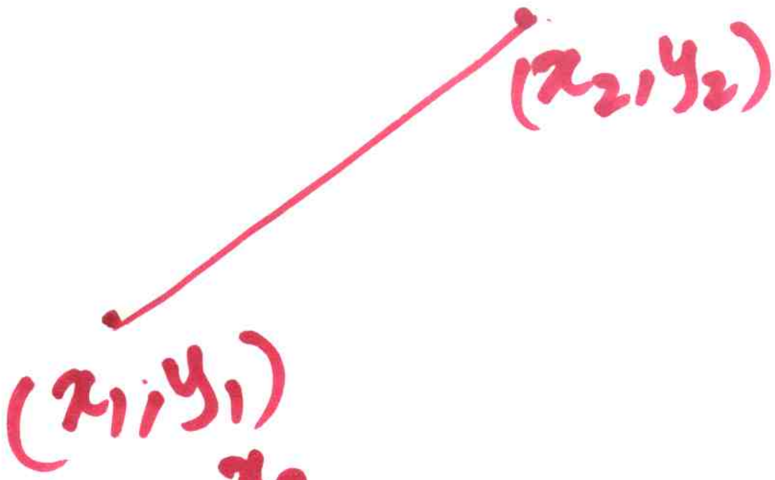
$$\frac{1}{2} m v^2$$

V: Potential energy $V(x)$

$$mgh.$$

$$\mathcal{L} = \int_{t_1}^{t_2} \left\{ \frac{1}{2} m \dot{x}^2 - V(x) \right\} dt$$

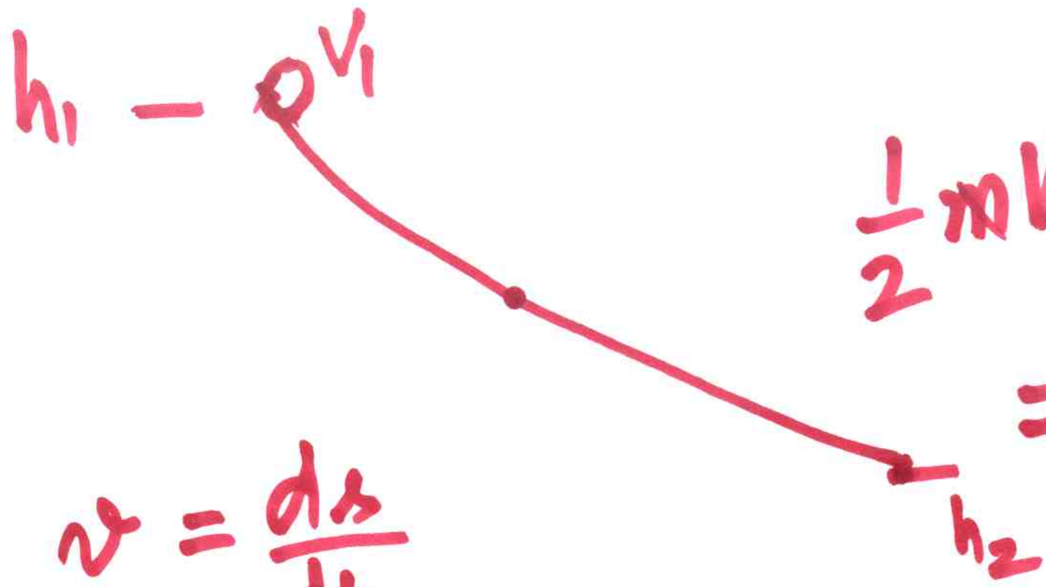
$$\mathcal{L} = \int_{t_1}^{t_2} F\left(t, x, \frac{dx}{dt} \equiv \dot{x}\right) dt$$



$$I = \int_{x_1}^{x_2} \sqrt{dx^2 + dy^2}$$

$$= \int_{x_1}^{x_2} \sqrt{1 + y'^2}\, dx$$

$$F\left(x, y, \frac{dy}{dx}\right)$$



$$\frac{1}{2} m v_1^2 - mgh_1 = \frac{1}{2} m v^2 - \cancel{m}gh$$

$$v = \sqrt{v_1^2 - 2g(h_1 - h)}$$

$$v = \frac{ds}{dt}$$

$$dt = \frac{ds}{v}$$

$$I = \int_{(1)}^{(2)} \frac{ds}{v} = \int^{2} \frac{\sqrt{dx^2 + dy^2}}{v}$$

$$= \int_1^2 \frac{\sqrt{1 + y'^2}}{v}\, dx$$

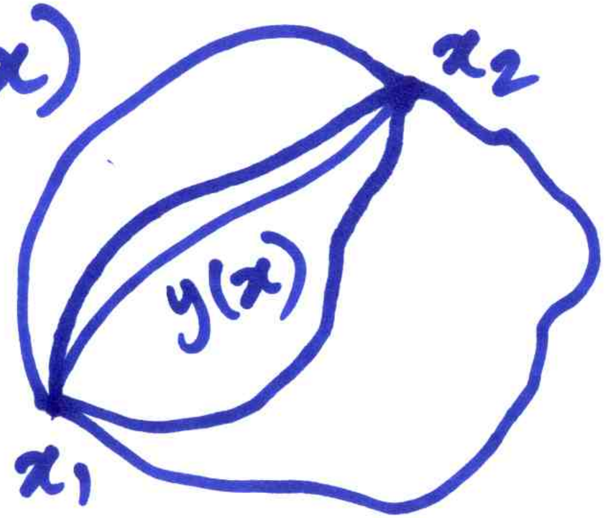
$$I = \int_{x_1}^{x_2} \frac{\sqrt{1+y'^2}}{\sqrt{V_1^2 - 2g(h_1 - h)}}\, dx .$$

$$I = \int_{x_1}^{x_2} F\left(x, y, y' \equiv \frac{dy}{dx}\right) dx$$

$a \rightarrow f(x)$

$y(x) \longrightarrow$ Extremises I

$$\underbrace{\tilde{y}(x)}_{\text{Admissible functions}} = y(x) + \varepsilon\, \eta(x)$$



$$\eta(x_1) = \eta(x_2) = 0$$

$$\tilde{I} = \int_{x_1}^{x_2} F(x, \tilde{y}, \tilde{y}')\, dx$$

$$\tilde{\hat{I}} = \int F(x, \tilde{y}, \tilde{y}')\,dx$$

$$= \int F(x, y+\varepsilon\eta, y'+\varepsilon\eta')\,dx$$

$$\tilde{\hat{I}} \simeq \left(\tilde{\hat{I}}\right)_{\varepsilon=0} + \left(\frac{d\tilde{I}}{d\varepsilon}\right)_{\varepsilon=0} \varepsilon + \left(\frac{d^2\tilde{I}}{d\varepsilon^2}\right)_{\varepsilon=0} \frac{\varepsilon^2}{2} + \cdots$$

$$\delta I = \tilde{\hat{I}} - I = \left(\frac{d\tilde{I}}{d\varepsilon}\right)_{\varepsilon=0} \varepsilon + \frac{d^2\tilde{I}}{d\varepsilon^2} \frac{\varepsilon^2}{2!} + \cdots$$

$$\left(\frac{d\tilde{I}}{d\varepsilon}\right)_{\varepsilon=0} \equiv 0$$ Necessary condition.

$$\left.\int_{x_1}^{x_2}\left[\frac{\partial F}{\partial \tilde{y}}\frac{d\tilde{y}}{d\varepsilon}+\frac{\partial F}{\partial \tilde{y}'}\frac{d\tilde{y}'}{d\varepsilon}\right]dx\right|_{\varepsilon=0}=0$$

$$\int_{x_1}^{x_2}\left[\frac{\partial F}{\partial y}\eta+\frac{\partial F}{\partial y'}\eta'\right]dx=0$$

$$\int_{x_1}^{x_2}\left[\frac{\partial F}{\partial y}\eta-\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)\eta\right]dx+\cancel{\left.\frac{\partial F}{\partial y'}\eta\right|_{x_1}^{x_2}}$$

$$\eta(x_1)=\eta(x_2)=0$$

$$\int_{x_1}^{x_2} \left[\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'} \right) \right] \eta \, dx = 0$$

$$\boxed{\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'} \right) \equiv 0}$$

Euler-Lagrange equation