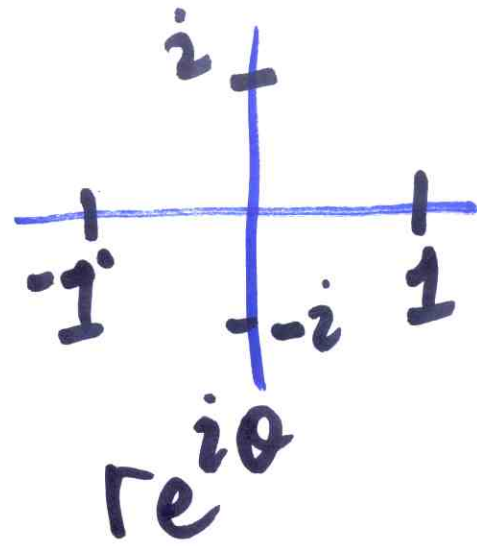


$$\left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\} + n \text{ mod } 2\pi$$

mod. 2π	+	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
0		0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\frac{\pi}{2}$		$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	0
π		π	$\frac{3\pi}{2}$	0	$\frac{\pi}{2}$
$\frac{3\pi}{2}$		$\frac{3\pi}{2}$	0	$\frac{\pi}{2}$	π

$$e^{i2\pi}, e^{i\frac{3\pi}{2}}, e^{i\pi}, e^{i\frac{\pi}{2}}$$



$$i, i^2 = -1, i^3 = -i, i^4 = 1.$$

$$\pm 1, \pm i$$

$$A, A^2, A^3, A^4 = 1.$$

Cyclic group of order 4

Order of a finite group — # of elements in the group

$$A, A^2, A^3, \dots, A^n = 1$$

$$\{1, -1, +i, -i\} \otimes = G$$

x	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

$$AI = IA = A$$

$$AA^{-1} = A^{-1}A = I$$

1) Closure.

2) Associativity.

3) Unit.

4) Inverse.

$$i(-i(1)) = (i(-i))1$$

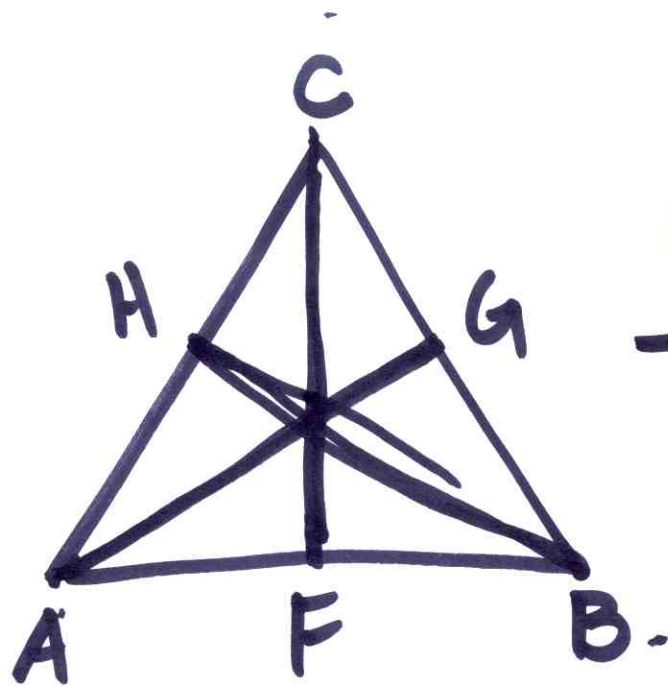
$$i(-i) = 1$$

$$-i^2 = 1$$

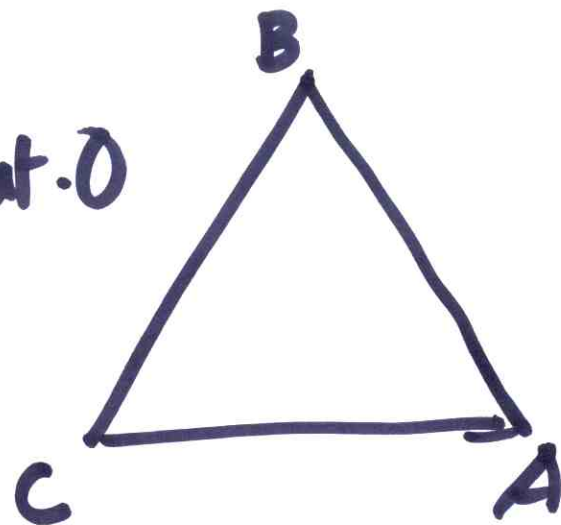
$$1 = 1$$

X	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

Isomorphic



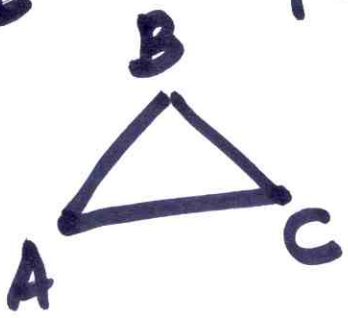
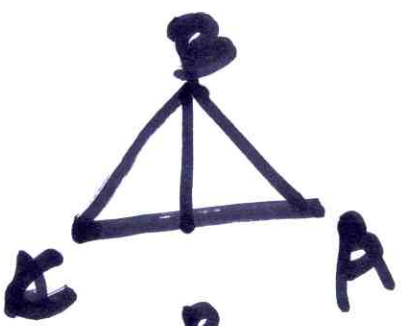
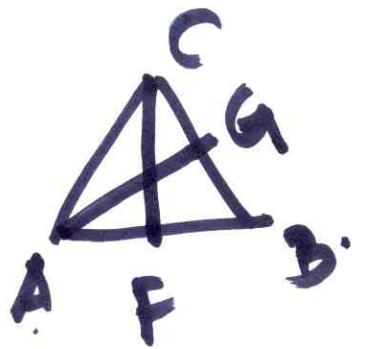
120° about O



$120^\circ, 240^\circ, 0^\circ$

$I = 0^\circ$
 $A = 120^\circ$
 $B = 240^\circ$

F
 G
 H.

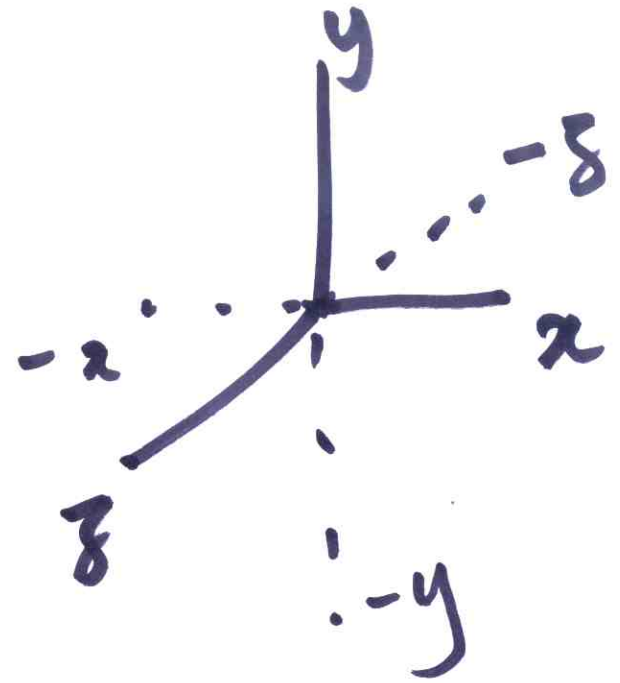


\oplus	I	A	B	F	G	H	→ Symmetry Operations
I	I	A	B	F	G	H	Symmetry Group of equilateral triangle.
A	A	B	I	G	H	F	
B	B	I	A	H	F	G	
F	F	H	G	I	B	A	
G	G	F	H	A	I	B	
H	H	G	F	B	A	I	

$$\begin{aligned}
 A(BG) &= (AB)G \\
 AF &= IG \\
 G &= G
 \end{aligned}$$

$$\underline{p} = \underline{T} \underline{q}$$

$$p_i = T_{ij} q_j$$



All second rank tensors

- Inversion symmetry