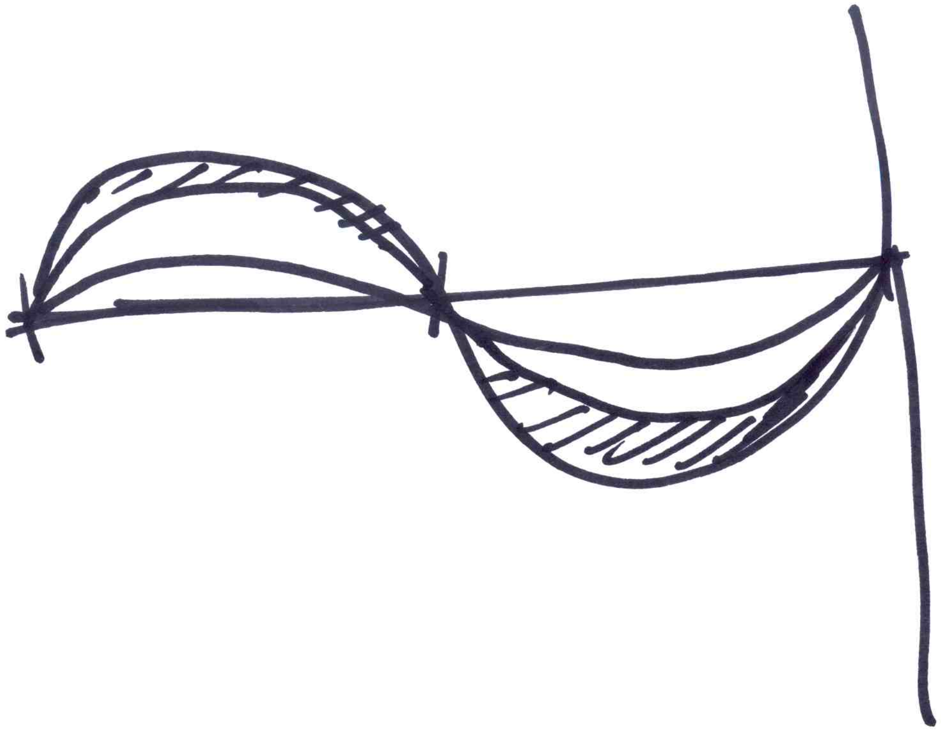
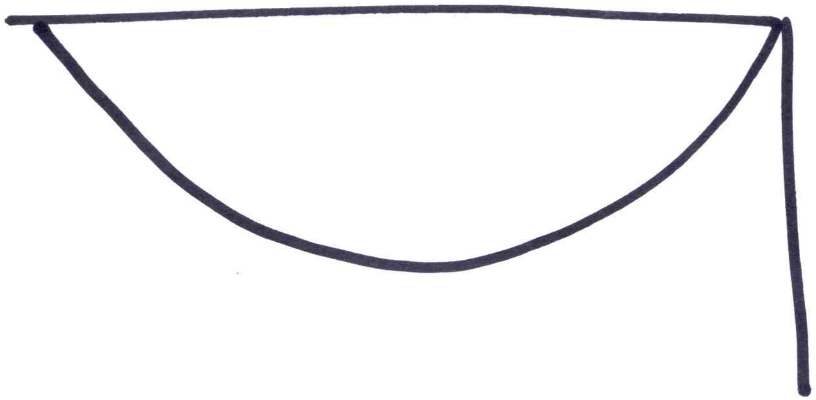
Prof. MP Gururajan
Lec. 42 to 45 Date 22/12/2015





$$\vec{J} = -\underline{\underline{D}}\,\underset{\sim}{\nabla c}$$

$\underbrace{\text{Vectors}}_{\text{I rank tensors}}$: $\vec{J}$, ∇c

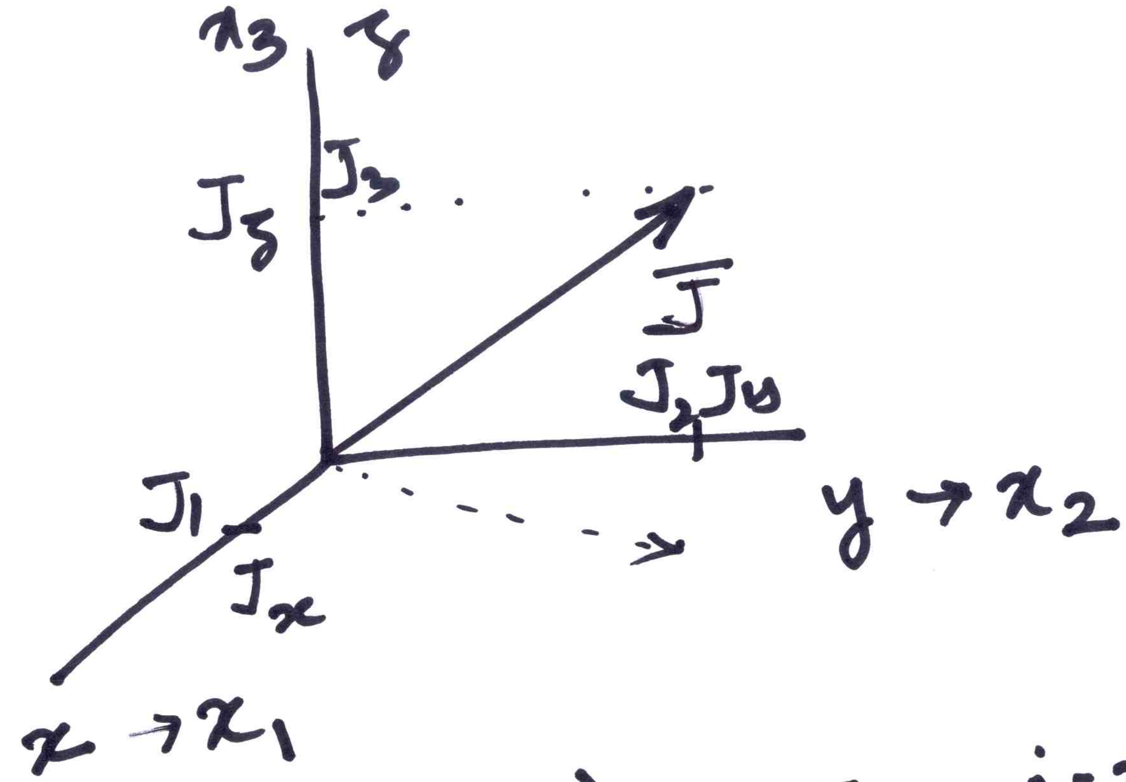
Quantity with magnitude and direction

$\underbrace{\text{Scalars}}_{\text{0th rank tensors}}$: Quantities with only magnitude

$\underline{\underline{c}} \rightarrow$ Scalar.

D : diffusivity II rank tensor.

$$\bar{J} = -D\,\nabla C$$



$$\bar{J} = (J_x, J_y, J_z) = J_i \qquad \begin{array}{l} i = x, y, z \\ i = 1, 2, 3 \end{array}$$

$$\bar{J} = \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

Scalars: Quantities with <u>no</u> indices.

Zeroth rank tensors

Vectors: Quantities with <u>one</u> index

First rank tensors

II rank tensors: Quantities with <u>two</u> indices

⋮

n^{th} rank tensor: Quantities with <u>n</u> indices

$$\vec{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = J_i$$

$$\nabla c = \begin{pmatrix} \frac{\partial c}{\partial x_1} \\ \frac{\partial c}{\partial x_2} \\ \frac{\partial c}{\partial x_3} \end{pmatrix} = \nabla_i c \qquad \nabla_i = \frac{\partial}{\partial x_i}$$

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = -\underline{\underline{D}} \begin{pmatrix} \frac{\partial c}{\partial x_1} \\ \frac{\partial c}{\partial x_2} \\ \frac{\partial c}{\partial x_3} \end{pmatrix}$$

$$\begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix} = - \underbrace{\begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}}_{\text{II rank tensor}} \begin{pmatrix} \frac{\partial c}{\partial x_1} \\ \frac{\partial c}{\partial x_2} \\ \frac{\partial c}{\partial x_3} \end{pmatrix}$$

Einstein Summation convention

$$J_i = D_{ij} \nabla_j c$$

Repeated indices are summed

$$\bar{J} = \underline{\underline{D}} \, \nabla c \qquad \sum_j$$

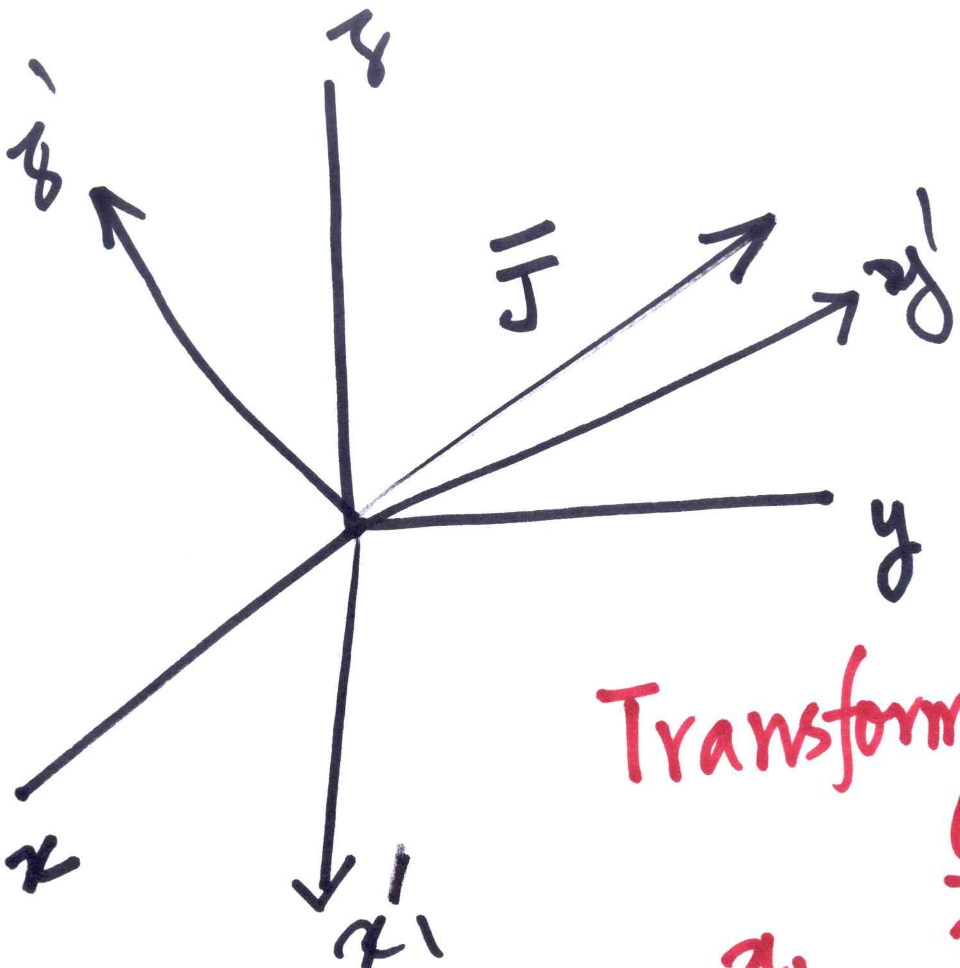
$$\mathbf{D} \quad \mathbb{D}$$

$$J_i = -D_{ik} \nabla_k c = -D_{i\alpha} \nabla_\alpha c$$

$$= -D_{ij} \nabla_j c$$

Dummy index

$$= -\cancel{D_{kk} \nabla_k c}$$



Transformation matrix

a_{ij} — Direction cosines for x_i' with x_j

$$
\begin{array}{cc|ccc}
 & & & \text{Old} & \\
 & & x_1 & x_2 & x_3 \\
\hline
\text{New} & x_1' & a_{11} & a_{12} & a_{13} \\
 & x_2' & a_{21} & a_{22} & a_{23} \\
 & x_3' & a_{31} & a_{32} & a_{33}
\end{array}
$$

$$x_i' = a_{ij} x_j$$

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\boxed{J_i' = a_{ij} J_j} \qquad \underline{x} \rightarrow \underline{x}'$$

$\vec{J}$ — is a vector.

Scalar — Invariant under coordinate transformation