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$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

(x, y, z)

(x_1, x_2, x_3)

$c(\underline{x}, t)$

Spatial Fourier transform

$\underline{x} \rightarrow \underline{k}$ wave vector

Reciprocal

$$c(\underline{k}, t) = A \int_{-\infty}^{\infty} c(\underline{x}, t) \exp(i \underline{k} \cdot \underline{x}) \, d\underline{x}$$

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

$$c(k,t) \equiv \tilde{c}$$

$$\frac{\partial \tilde{c}}{\partial t} = D \frac{\partial^2}{\partial x^2} \tilde{c}$$

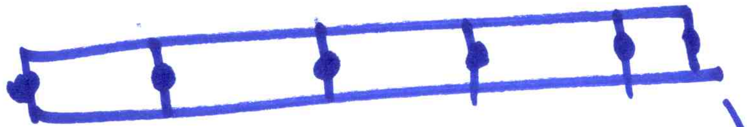
$$\frac{\partial}{\partial t} \int c \exp(i \underline{k} \cdot \underline{x}) \, d\underline{x} = D \frac{\partial^2}{\partial x^2} \int c \exp(i \underline{k} \cdot \underline{x}) \, dx$$

$$\boxed{\frac{d}{dt}\tilde{c} = -Dk^2\tilde{c}}$$ Kreyzig

Fast Fourier Transform

FFT

DFT - Discrete Fourier Transform



$$c(x,t) = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_{n-1} \end{pmatrix}_t = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}_t$$

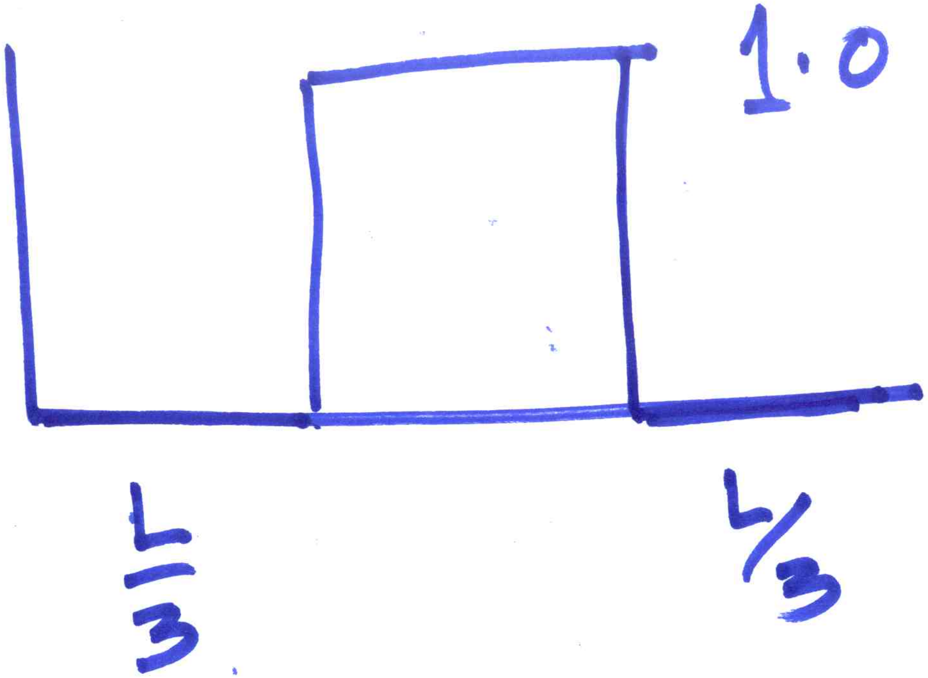
$$\frac{d\tilde{c}}{dt} = -Dk^2\tilde{c}$$

$$c \longrightarrow \tilde{c}$$

$$\frac{\tilde{c}^{t+\Delta t} - \tilde{c}^{t}}{\Delta t} = -Dk^2\tilde{c}^{t+\Delta t}$$

$$\tilde{c}^{t+\Delta t}(1+Dk^2\Delta t) = \tilde{c}^{t}$$

$$\boxed{\tilde{c}^{t+\Delta t} = \frac{\tilde{c}^{t}}{1+Dk^2\Delta t}}$$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3
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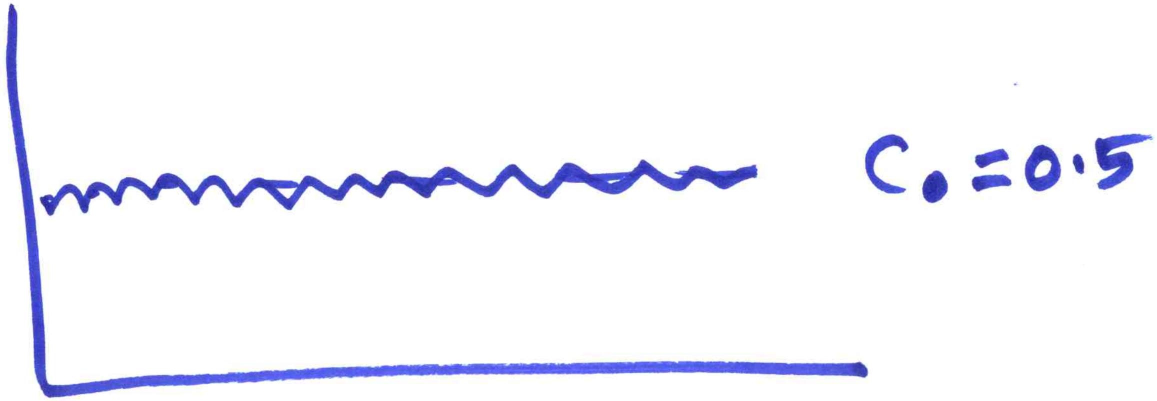
C₀ = 0.5

$$\tilde{c}^{\,t+\Delta t} = \frac{\tilde{c}^{\,t}}{1+D\underline{\underline{k}}^2\Delta t}$$

$$W \doteq 0 \Rightarrow W + n$$

$$e = n+1 \Rightarrow n - 1$$

$$\left[\begin{array}{cc} 1 & 1 \\ -\frac{1}{2} & \frac{1}{2} \\ -\frac{\pi}{a} & \frac{\pi}{a} \\ -\frac{L}{2} & \frac{L}{2} \end{array}\right.$$