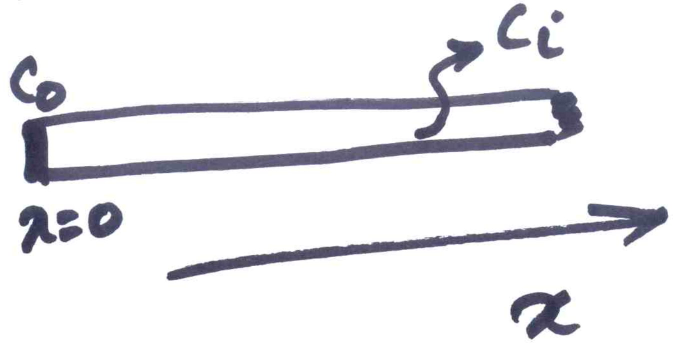
$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$



$$\eta = \frac{c - c_i}{c_0 - c_i} \qquad 0 \leq \eta \leq 1$$

t_0 - Characteristic time

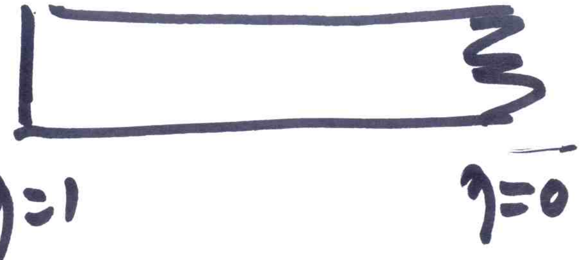
L - Characteristic length.

$$\frac{x}{L} = \xi$$

$$\frac{t}{t_0} = \tau$$

$$\frac{\partial \eta}{\partial \tau} = \frac{D t_0}{L^2} \cdot \frac{\partial^2 \eta}{\partial \xi^2}$$

$$t_0 = \frac{L^2}{D}$$



$$\frac{D t_0}{L^2} = 1.$$

$$\boxed{\frac{\partial \eta}{\partial \tau} = \frac{\partial^2 \eta}{\partial \xi^2}}$$

$\eta = 0$ at $\tau = 0$ for $0 < \xi \leq 1$

$\eta = 1$ at $\tau > 0$ for $\xi = 0$

$$\eta = f\left(\frac{\xi}{\sqrt{\tau}}\right) \qquad \frac{x}{\sqrt{Dt}}$$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2} \qquad \boxed{\frac{\partial \eta}{\partial \tau} = \frac{\partial^2 \eta}{\partial \xi^2}}$$

$$\frac{x}{\sqrt{Dt}}$$

Non-dimensional quantity $= \dfrac{x^2}{Dt}$.

$x^* = \sqrt{Dt}$ Diffusion distance

$x^2 = Dt$.

$$\frac{\partial \eta}{\partial \tau} = \frac{\partial^2 \eta}{\partial \xi^2}$$

$$\eta = f\left(\xi / \sqrt{\tau} \right) \qquad \eta = f(y)$$

$$y = \xi / \sqrt{\tau}$$

$$\frac{d\eta}{dy} = p$$

$$\frac{\partial \eta}{\partial \tau} = \frac{\partial \eta}{\partial y} \cdot \frac{\partial y}{\partial \tau}$$

$$y = \xi/\sqrt{\tau} \qquad \frac{\partial y}{\partial \tau} = -\frac{1}{2}\frac{\xi}{\tau\sqrt{\tau}}$$

$$= -\frac{y}{2\tau}$$

$$\boxed{\frac{\partial \eta}{\partial \tau} = -\left(\frac{\partial \eta}{\partial y}\right)_{\xi} \frac{y}{2\tau}}$$

$$\frac{\partial^2 \eta}{\partial \xi^2} = \frac{\partial}{\partial \xi} \cdot \frac{\partial \eta}{\partial \xi}$$

$$= \frac{\partial}{\partial \xi} \cdot \frac{\partial \eta}{\partial y} \cdot \frac{\partial y}{\partial \xi}$$

$$y = \xi / \sqrt{c} \quad \Rightarrow \quad \frac{\partial y}{\partial \xi} = \frac{1}{\sqrt{c}}$$

$$= \frac{\partial}{\partial \xi} \cdot \frac{\partial \eta}{\partial y} \cdot \frac{1}{\sqrt{c}}$$

$$= \frac{1}{\sqrt{c}} \cdot \frac{\partial}{\partial \xi} \left(\frac{\partial \eta}{\partial y} \right)$$

$$\frac{\partial^2 \eta}{\partial \xi^2} = \frac{1}{\sqrt{\tau}} \cdot \frac{\partial}{\partial \xi}\left(\frac{\partial \eta}{\partial y}\right)$$

$$= \frac{1}{\sqrt{\tau}} \frac{\partial}{\partial y}\left(\frac{\partial \eta}{\partial y}\right) \cdot \frac{\partial y}{\partial \xi}$$

$$= \frac{1}{\tau} \cdot \left(\frac{\partial^2 \eta}{\partial y^2}\right)_\tau$$

$$\frac{\partial \eta}{\partial \tau} = -\frac{y}{2\tau} \cdot \left(\frac{\partial \eta}{\partial y}\right)$$

$$-\frac{y}{2\tau}\cdot\frac{d\eta}{dy} = \frac{1}{\tau}\cdot\frac{d^2\eta}{dy^2}$$

$$\frac{d\eta}{dy} = P.$$

$$\boxed{\frac{dP}{dy} = \frac{-yP}{2}} \quad \text{ODE}$$

$$\frac{dP}{P} = \frac{-y\,dy}{2}$$

$$\ln P = \frac{-y^2}{4} + \ln A$$

$$P = A \exp\left(-\frac{y^2}{4}\right)$$

$$\frac{d\eta}{dy} = A \exp\left(-\frac{y^2}{4}\right)$$

$$\boxed{\eta = \int A \exp\left(-\frac{y^2}{4}\right) dy}$$

$$\frac{c - c_i}{c_0 - c_i} = \int A \exp\left(-\frac{y^2}{4}\right) dy$$

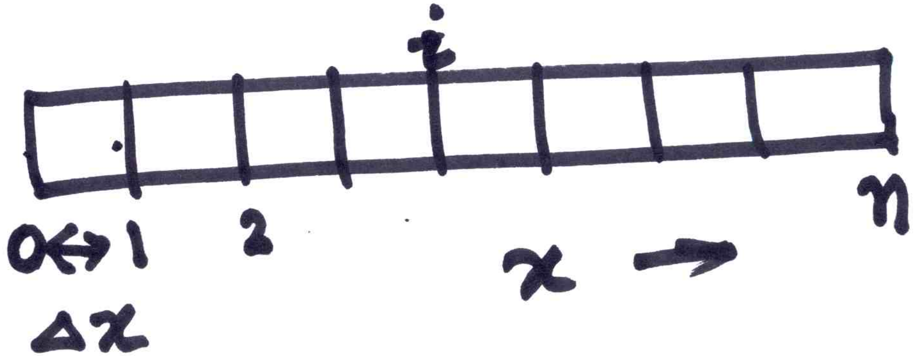
$$y = \xi / \sqrt{\tau} \qquad y^2 = \frac{\xi^2}{1\tau} \qquad x^2 = Dt$$

$$C(x,t) = c_i + (c_0 - c_i) \int A \exp\left(-\frac{y^2}{4}\right) dy$$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

Finite difference ✓

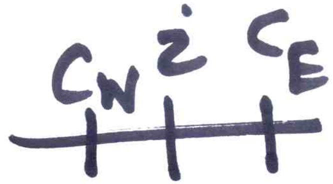
Spectral technique - Fourier



$$\frac{L}{\Delta x} = n$$

$$\left(\frac{\partial c}{\partial t}\right)_i \approx \frac{c_i^{t+\Delta t} - c_i^{t}}{\Delta t}$$

$$D\frac{\partial^2 c}{\partial x^2} = \frac{D}{(\Delta x)^2}\left(c_W^t + c_E^t - 2c_i^t\right)$$



Explicit

$c_W = i-1$

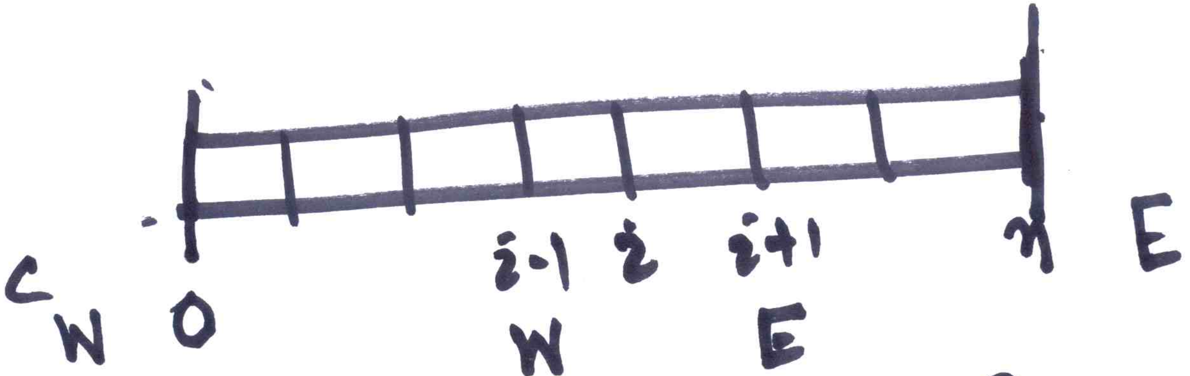
$c_E = i+1$

$$\frac{c_i^{t+\Delta t} - c_i^t}{\Delta t} = \frac{D}{(\Delta x)^2}\left(c_{i-1}^t + c_{i+1}^t - 2c_i^t\right)$$

$$\boxed{c_i^{t+\Delta t} = c_i^t(1-2\alpha) + \alpha\left(c_{i-1}^t + c_{i+1}^t\right)}$$

Where $\alpha = \dfrac{D\Delta t}{(\Delta x)^2}$ $\quad t=0 \quad c_i$

Initial condition



if $i=0$, what is c_W?
if $i=n$, what is c_E?
2 Boundary conditions

	$i=0$	$i=n$
Dirichlet –	$c=c_0$	$c=c_0$
Neumann	$\left(\frac{\partial c}{\partial x}\right)=\alpha$	$\left(\frac{\partial c}{\partial x}\right)=\alpha$
Robin	D + N	D+N.

$$D\frac{\partial^2 C}{\partial x^2} = \frac{D}{(\Delta x)^2}\cdot\left[C_W^{t+\Delta t} + C_E^{t+\Delta t} - 2C_i^{t+\Delta t}\right]$$

$$\frac{C_i^{t+\Delta t} - C_i^{t}}{\Delta t} = \frac{D}{(\Delta x)^2}\left[C_W^{t+\Delta t} + C_E^{t+\Delta t} - 2C_i^{t+\Delta t}\right]$$

$$C_i^{t+\Delta t}(1+2\alpha) + \alpha\left[C_W^{t+\Delta t} + C_E^{t+\Delta t}\right] = C_i^{t}$$

$$\begin{bmatrix} 1+2\alpha & \alpha & & & \\ \alpha & 1+2\alpha & \alpha & & \\ & \alpha & 1+2\alpha & \alpha & \\ & & & & \cdot \end{bmatrix} \begin{bmatrix} C_0^{t+\Delta t} \\ C_1^{t+\Delta t} \\ \vdots \\ C_n^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} C_0^{t} \\ C_1^{t} \\ \vdots \\ C_n^{t} \end{bmatrix}$$