Constitutive laws; Ficks I law $\quad J = -D\nabla C$

$q = -k\nabla T$ Fourier law of heat conduction

Conservation law $\quad \frac{\partial C}{\partial t} = -\nabla \cdot J$ Mass

$$\boxed{\frac{\partial C}{\partial t} = D\nabla^2 C}$$

Energy

$$\frac{\partial U}{\partial t} = -\nabla \cdot q$$

$$\frac{\partial U}{\partial t} = k\nabla^2 T$$

$$U = \rho C_p T$$

$$\frac{k}{\rho C_p} = \frac{1}{\alpha}$$

$$\boxed{\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\nabla^2 T}$$

$$\frac{\partial c}{\partial t'} = D' \frac{\partial^2 c}{\partial x'^2}$$

– primed quantities – Non-dimensional

$$\boxed{\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}}$$

t, x, D — Non-dimensional.

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

Boundary conditions

$$c(0,t) = 0$$

$$c(L,t) = 0$$

Initial condition

$$c(x,0) = f(x) \text{ with}$$

$$f(0) = f(L) = 0$$

$$c(x,t) = F(x)\,G(t)$$

$$\frac{\partial c}{\partial t} = D\,\frac{\partial^2 c}{\partial x^2}$$

$$\frac{\partial (FG)}{\partial t} = D\,\frac{\partial^2 (FG)}{\partial x^2}$$

$$F\,\frac{\partial G}{\partial t} = DG\,\frac{\partial^2 F}{\partial x^2}$$

$$\frac{\dot{G}}{DG} = \frac{1}{F}F'' = -p^2 \text{ (say)}$$

$$\frac{\dot{G}}{DG} = -p^2$$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

$$c = F(x)G(t)$$

$$\frac{F''}{F} = -p^2$$

$$\boxed{\dot{G} + p^2 DG = 0}$$ ODE 1

$$\boxed{F'' + p^2 F = 0}$$ ODE 2.

$$F'' + p^2 F = 0$$

$$F(x) = A\cos px + B\sin px$$

$$C(0,t) = 0 \qquad C = FG$$

$$F(0)\,G(t) = 0$$

$$C(L,t) = 0$$

$$F(L)\,G(t) = 0$$

$$F(0) = 0 \qquad F(L) = 0$$

$$F(0) = 0 = A \Rightarrow \boxed{A=0}$$

$$F(L) = 0 = B\sin pL = 0$$

$$\sin pL = 0$$

$$pL = n\pi \qquad n = 1, 2, \ldots$$

$$\boxed{p = \frac{n\pi}{L}} \qquad n = 1, 2, \ldots$$

$$\boxed{F_n(x) = B_n \sin \frac{n\pi x}{L}} \qquad n = 1, 2, \ldots$$

$$\dot{G} + p^2 D G = 0$$

$$p^2 = \frac{n^2\pi^2}{L^2}$$

$$\dot{G} + \frac{n^2\pi^2}{L^2} D \, G = 0$$

$$\boxed{\dot{G} + \lambda_n^2 G = 0} \quad \text{with } \lambda_n = \frac{n\pi}{L} \cdot \sqrt{D}$$

$$G_n(t) = k_n \exp(-\lambda_n^2 t)$$

$$C(x,t) = F(x)\,G(t)$$

$$= k_n \exp(-\lambda_n^2 t) \cdot B_n \cdot \sin \frac{n\pi x}{L}.$$

$$C_n(x,t) = A \exp(-\lambda_n^2 t) \sin \frac{n\pi x}{L}.$$

$$C = \sum c_n(x,t)$$

$$C(x,t) = \sum_{n=1}^{\infty} A_n \exp(-\lambda_n^2 t) \sin \frac{n\pi x}{L}.$$

$n = 1, 2, \ldots$

$$C(x,0) = f(x)$$

$$f(x) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{L}.$$

$$A_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} \, dx$$

$$n = 1, 2 \ldots$$

$$C(x,t) = \sum_{n=1}^{\infty} A_n \exp(-\lambda_n^2 t) \sin \frac{n\pi x}{L}.$$

$$c - c_0 = A(\beta, t)\exp(i\beta x)$$

$$c(x,t) = \sum_{n=1}^{\infty} A_n \exp(-\lambda_n^2 t)\,\sin\frac{n\pi x}{L}$$