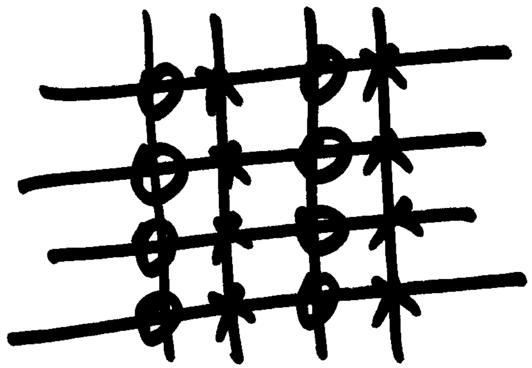
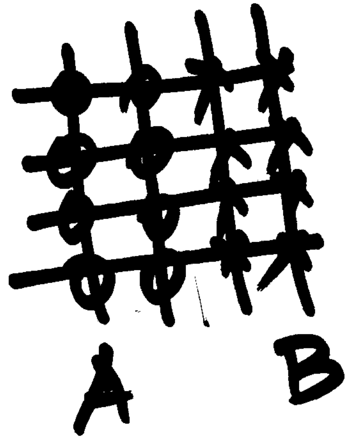
9/12/2015

$$\frac{\partial c}{\partial t} = \underbrace{\frac{M}{N_v} G''}_{D.} \frac{\partial^2 c}{\partial x^2}$$

Fick's II law
Classical diffusion equation.





$$\mu_B = \frac{\partial G}{\partial n_B}$$

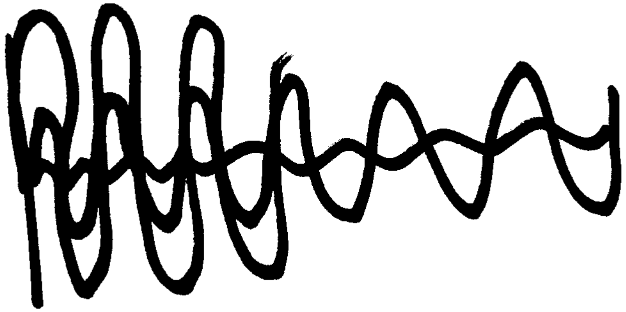
AB

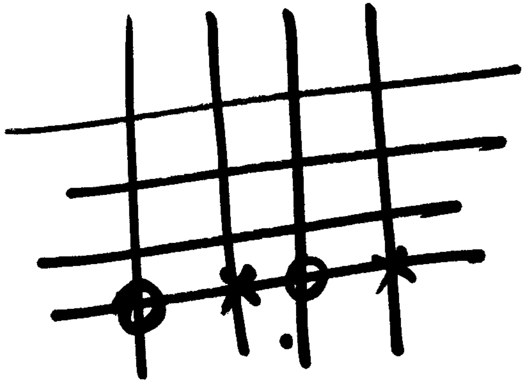
Interface

$$\boxed{\frac{\partial c}{\partial t} = \frac{M}{N_v}\left[G'' \frac{\partial^2 c}{\partial x^2} - 2K \frac{\partial^4 c}{\partial x^4}\right]}$$

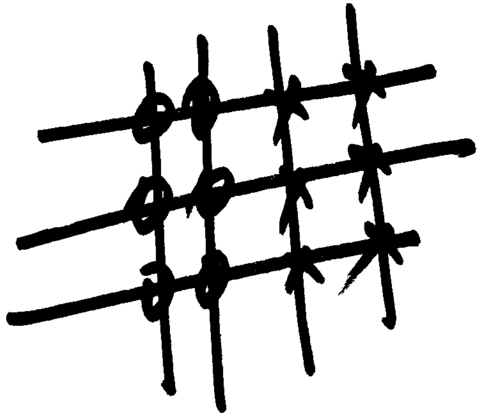
Gradient energy coefficient

Non-Classical diffusion
Cahn-Hilliard equation





Ordered



$\lambda \simeq 100\,\text{Å}$

Cu-Ni-Fe

$\Omega > 0.$

$G'' < 0.$

$$\boxed{\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}}$$

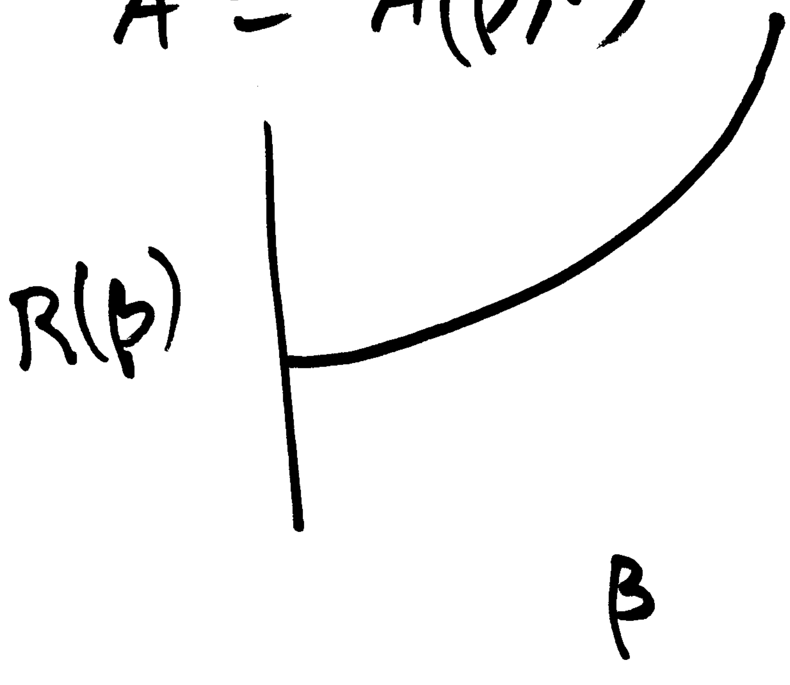
$D < 0.$

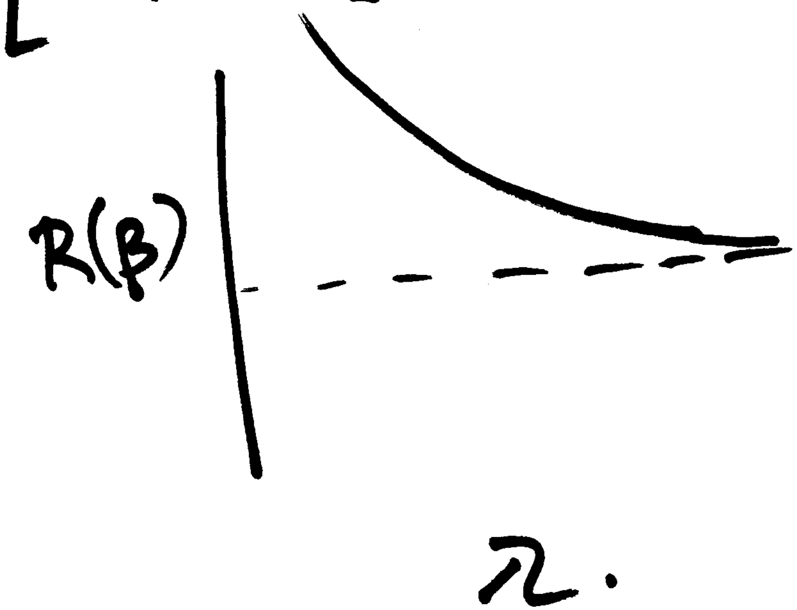
$$R(\beta) = -\frac{M}{N_v} G'' \beta^2 .$$

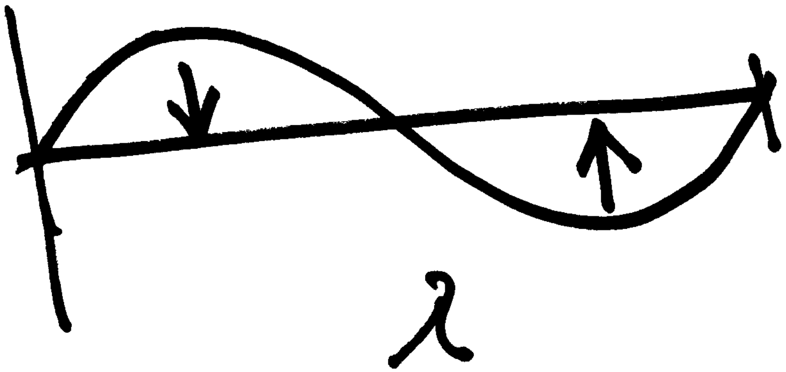
if $G'' < 0$,

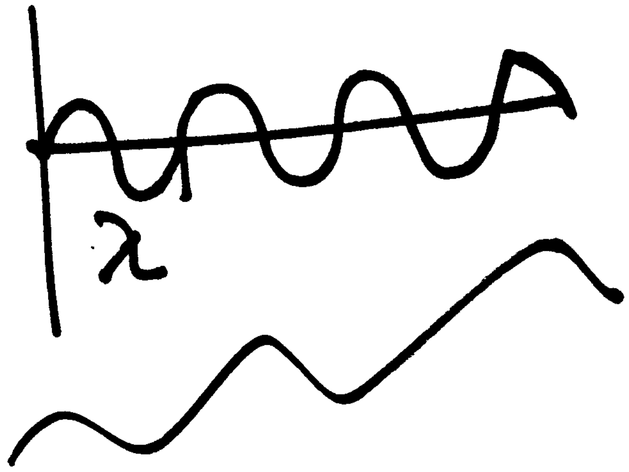
$R(\beta) > 0$.

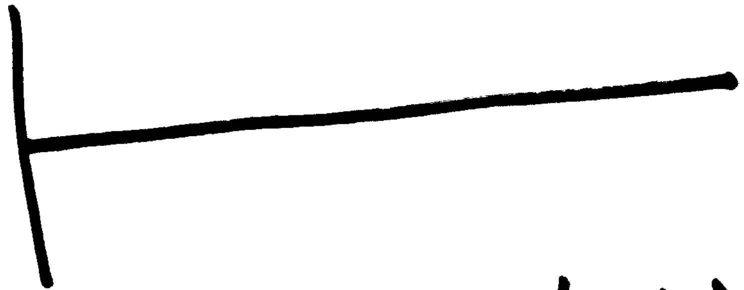
$$A = A(\beta, 0) \exp\left[R(\beta) t \right]$$











$$\beta = \frac{2\pi}{\lambda}$$

$$A = A(\beta, 0)\, \exp\big(\underbrace{R(\beta)}_{<0 \text{ if } G'' > 0.}\, t\big)$$

$\Rightarrow$ Homogenisation (No nucleation)

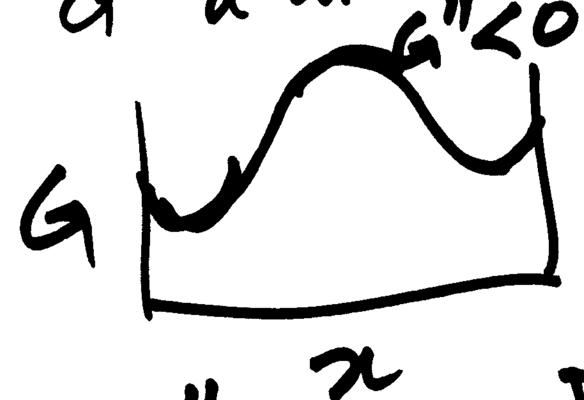
$$J = -MG'' \frac{\partial c}{\partial x}.$$

$$\frac{\partial c}{\partial t} = + \frac{1}{N_v} \frac{\partial}{\partial x} \left(+MG'' \frac{\partial c}{\partial x} \right)$$

$$\frac{\partial c}{\partial t} = \frac{M}{N_v} G'' \frac{\partial^2 c}{\partial x^2}$$

assuming M, G'' a constant

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}.$$



$$D = \frac{M}{N_v} G'' \Rightarrow G''<0, \text{ then } D<0$$

$$N_V(\mu_2 - \mu_1) = \frac{\partial G}{\partial c}$$

$$J = -N_V\, M \left[\frac{\partial \mu_2}{\partial x} - \frac{\partial \mu_1}{\partial x}\right]$$

$$= -\cancel{N_V}\, M \frac{\partial}{\partial x}\left(\frac{\partial G}{\partial c}\right) \qquad \frac{\partial}{\partial x} = \frac{\partial}{\partial c} \cdot \frac{\partial c}{\partial x}$$

$$= -M \frac{\partial^2 G}{\partial c^2} \cdot \frac{\partial c}{\partial x}$$

$$\boxed{J = -M\, G'' \frac{\partial c}{\partial x}} \qquad \frac{\partial c}{\partial t} = -\frac{1}{N_V} \cdot \frac{\partial J}{\partial x}.$$

$$J = -N_v\, c(1-c)\left[v_2 \frac{\partial \mu_2}{\partial x} - v_1 \frac{\partial \mu_1}{\partial x}\right]$$

$$= -N_v\, c(1-c)\left\{(1-c)v_2 + c\, v_1\right\}\left[\frac{\partial \mu_2}{\partial x} - \frac{\partial \mu_1}{\partial x}\right]$$

$$+ (v_2 - v_1)\left[c\frac{\partial \mu_2}{\partial x} + (1-c)\frac{\partial \mu_1}{\partial x}\right]$$

$\equiv 0$ Gibbs-Duhem relationship.

$$= -N_v\, M\left[\frac{\partial \mu_2}{\partial x} - \frac{\partial \mu_1}{\partial x}\right]$$

Where $M \equiv c(1-c)\left\{(1-c)v_2 + c\, v_1\right\}$.

$$J = J_2 - c(J_1 + J_2)$$

$$= -N_v c\, v_2 \frac{\partial \mu_2}{\partial x} + c N_v (1-c)\, v_1 \frac{\partial \mu_1}{\partial x}$$

$$+ c N_v c\, v_2 \frac{\partial \mu_2}{\partial x}$$

$$= -N_v c \left[v_2 \frac{\partial \mu_2}{\partial x} - c\, v_2 \frac{\partial \mu_2}{\partial x} - (1-c)\, v_1 \frac{\partial \mu_1}{\partial x} \right]$$

$$J = -N_v c (1-c) \left[v_2 \frac{\partial \mu_2}{\partial x} - v_1 \frac{\partial \mu_1}{\partial x} \right]$$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

$$c - c_0 = A(\beta, t)\exp(i\beta x)$$

$$A = A(\beta, 0)\exp(-D\beta^2 t)$$

$$D = \frac{M}{N_V}G''$$

$$A = A(\beta, 0)\exp\left(-\frac{M}{N_V}G''\beta^2 t\right)$$

$$A = A(\beta, 0)\exp(R(\beta)t) \quad R(\beta) = -\frac{M}{N_V}G''\beta^2$$

$$\frac{\partial c}{\partial t} -$$

$J = -D\nabla c$ — Fick's I law

$$J_1 = -N_v(1-c)\,v_1 \frac{\partial \mu_1}{\partial x}$$

μ_1 → Chemical potential per atom

v_1 → Velocity of atoms (of component 1) under unit potential gradient

$$J_2 = -N_v\,c\,v_2 \frac{\partial \mu_2}{\partial x}$$

Matano interface — Plane - moving
total flux $\equiv 0$