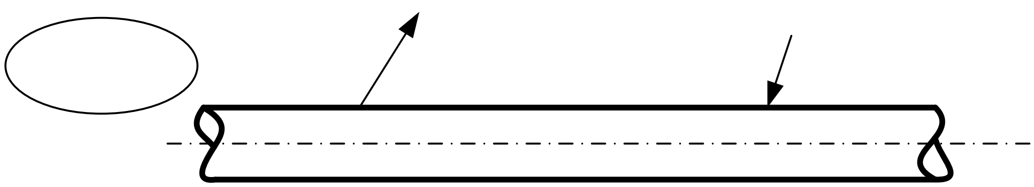
Module 8: Solved Problems

1. Electric current passes through a long wire of diameter 1-mm, and dissipates 3150W/m. The wire reaches surface temperature of 126°C when submerged in water at 1 bar. Calculate the boiling heat transfer coefficient and find the value of the correlation coefficient $C_{s,f}$.

   Known: long wire, 1–mm-diameter, reaches a surface temperature of 126°C in water at 1atm while dissipating 3150W/m.

   Find: (1) Boiling heat transfer coefficient and (2) correlation coefficient $C_{s,f}$, if nucleate boiling occurs

   Schematic:



   Assumptions: (1) Steady-state conditions, (2) Nucleate boiling.

   From property table: Water (saturated, 1atm) T_s=100C, $\rho_l=1/v_f=957.9 kg/m^3$, $\rho_f=1/v_g=0.5955 kg/m^3$, $c_{p,l}=4217$ J/kg.K, $\mu_l=279*10^{-6}$ N.s/m², $p_{rl}=1.76$, $h_{fg}=2257$ KJ/kg, $\sigma=58.9*10^{-3}$N/m.

   Analysis: (a) For the boiling process, the rate equation can be rewritten as

$$\bar{h} = \frac{q_s''}{(T_s - T_{sat})} = \frac{q_s'}{\pi D}/(T_s - T_{sat})$$

$$\bar{h} = \frac{3150 W/m}{\pi \times 0.001 m}/(126-100)°C = 1.00 \times 10^6 \frac{W}{m^2}/26^\circ C = 38{,}600 W/m^2.K$$

q’=

Water

1 atm

Note that heat flux is very close to q"$_{max}$, and nucleate boiling exists.

(b) For nucleate boiling, the Rohsenow correlation may be solved for $C_{s,f}$, to give

$$\mathrm{C}_{s,f} = \left\{ \frac{\mu_\lambda h_{f,g}}{q_s^{'}} \right\}^{\frac{1}{3}} \left[\frac{g(\rho_\lambda - \rho_v)}{\sigma} \right]^{1/6} \left[\frac{c_{p,\lambda} \Delta T_e}{h_{f,g} \, \mathrm{Pr}_\lambda^n} \right]$$

Assuming the liquid surface combination is such that n=1 and substituting numerical values with $\Delta T_e = T_s - T_{sat}$, find

$$C_{s,f} = \left\{ \frac{279 \times 10^{-6} N.s/m \times 22257 \times 10^3 J/kg}{1.00 \times 10^6 W/m^2} \right\}^{1/3} \left[\frac{9.8 m/s^2 (957.9 - 0.5955 kg/m^3}{58.9 \times 10^{-3} N/m} \right]^{1/6}$$

$$\times \left(\frac{4217 J/kg.K \times 26K}{2257 \times 10^3 J/kg \times 1.76} \right)$$

$$C_{s,f} = 0.017$$

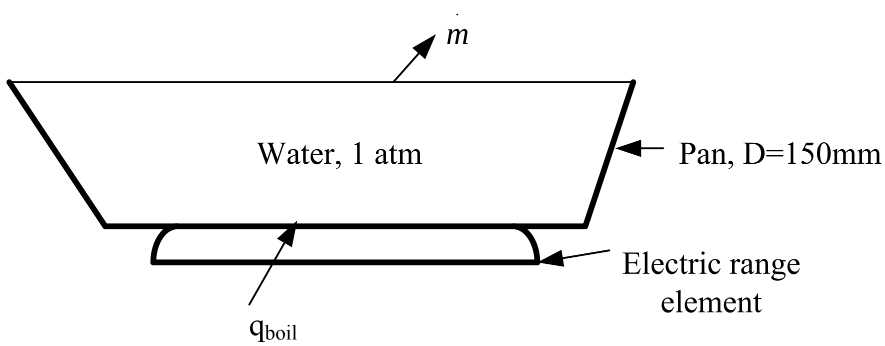
Comments: By comparison with the values $C_{s,f}$ for other water-surface combinations (given in standard tables), the $C_{s,f}$ value for the wire is quite large suggesting that its surface must be highly polished. Note that the value of the boiling heat transfer coefficient is much larger that for other convection processes previously encountered.

2. A copper pan has a diameter of 150 mm. The bottom of the pan is maintained at 115°C when placed on an electric cooking range. Estimate the heat required to boil the water in this pan. Determine the evaporation rate. What is the ratio of the surface heat flux to the critical heat flux? What pan temperature is required to achieve the critical heat flux?

   Known: copper pan 150 mm in diameter and filled with water at 1atm, maintained at 115°C.

   Find: the power required to boil the water and the evaporation rate; ratio of the heat flux to the critical heat flux; pan temperature is required to achieve the critical heat flux.

   Schematic:



   Assumptions: (1) Nucleate pool boiling, (2) Copper pan is polished surface.

   Properties: Table: Water (1atm) T_{sat}=100°C, ρ_l=957.9kg/m^3, ρ_v=0.5955kg/m^3, $c_{p,l}$=4217 J/kg.K, μ_l=279*10^{-6}N.s/m^2, p_{rl}=1.76, h_{fg}=2257 KJ/kg, σ=58.9*10^{-3}N/m.

   Analysis: the power requirement for boiling and the evaporation rate can be expressed as follows,

$$q_{boil} = q_s''.A_s \qquad\qquad \dot{m} = q_{boil} / h_{f,g}$$

The heat flux for nucleate pool boiling can be estimated using the Ronsenow Correlation.

$$q''_s = \mu_\lambda h_{f,g}\left[\frac{g(\rho_\lambda-\rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{p,\lambda}\Delta T_e}{C_{s,f}h_{f,g}\,\mathrm{Pr}_\lambda^n}\right]^3$$

Selecting $C_{s,f}$=0.013 and n=1 from standard table for the polished copper finish, find

$$C_{s,f} = \left\{279\times10^{-6}N.s/m\times2257\times10^3 J/kg\right\}\left[\frac{9.8m/s^2(957.9-0.5955kg/m^3}{58.9\times10^{-3}N/m}\right]^{1/6}$$
$$\times\left(\frac{4217J/kg.K\times26K}{2257\times10^3 J/kg\times1.76}\right)$$

$$C_{s,f} = 4.619\times10^5 W/m^2$$

The power and evaporation rate are

$$q_{boil} = 4.619\times10^5 W/m^2\times\frac{\pi}{4}(0.150m)^2 = 8.16kW$$

$$\dot{m}_{boil} = 8.16kW/2257\times10^3 J/kg = 3.62\times10^{-3}kg/s = 13kg/h$$

The maximum or critical heat flux was found as

Q''_{max}=1.26MW/m^2.

Hence, the ratio of the operating to maximum heat flux is

$$\frac{q''_s}{q''_{\max}} = 4.619\times10^5 W/m^2/1.26MW/m^2 = 0.367$$

From the boiling curve, $\Delta T_e \approx 30°C$ will provide the maximum heat flux

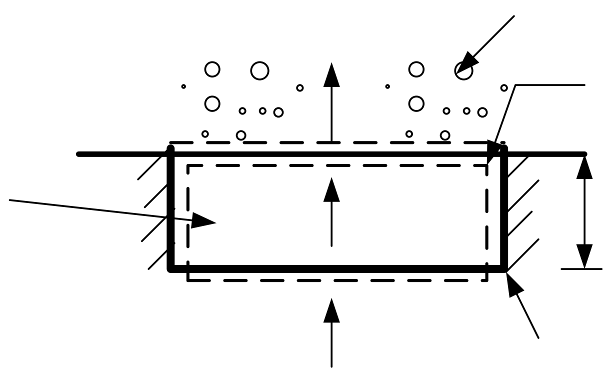
3. A silicon chip has a thickness L=25 mm and has thermal conductivity k_s=135W/m.K. The chip is cooled by boiling a saturated fluorocarbon liquid (T_{sat}=57°C) on its surface. The electronic circuits on the bottom of the chip are perfectly insulated.

   Properties of the saturated fluorocarbons are $c_{p,l}$=110J?kg.K, h_{fg}=84,400J/kg, ρ_l=1619.2kg/m^3, ρ_v=13.4kg/m^3,σ=8.1*10^{-3}kg/s^2,μ_l=440*10^{-6}kg/m.s and p_{rl}=9.01. The nucleate boiling constants are $C_{s,f}$=0.005 and n=1.7. What is the steady-state temperature at the bottom of the chip? If the chip bottom heat flux is raised to 90% of the critical heat flux, what is the new steady-state value of the chip bottom temperature?

   Known: Thickness and thermal conductivity of a silicon chip. Properties of saturated fluorocarbon liquid on top side.

   Find: (a) Temperature at bottom surface of chip for a prescribed heat flux, (b) Temperature of bottom surface at 90% of CHF.

   Schematic:



   Assumptions: (1) steady-state conditions, (2) uniform heat flux and adiabatic sides, hence one-dimensional conduction in chip, (3) Constant properties, (4) Nucleate boiling in liquid.

Properties: Saturated fluorocarbon (given): $c_{p,\lambda} = 1100 J/kg.K$, $h_{f,g}$=84,400 J/kg, ρ_λ=1619.2kg/m3, ρ_v=13.4kg/m3,σ=8.1*10^{-3}kg/s^2, μ_λ=440*10-6kg/m-s, $P_{r\wp}$=9.01.

Analysis: (a) Energy balances yield $q_o^" = q_{cond}^" = k_s(T_o - T_s)/L = q_b^"$.

Obtain T_s from the Rohsenow correlation.

$$T_s - T_{sat} = \frac{C_{s,f} h_{f,g} \Pr_\lambda^n}{c_{p,\lambda}} \left(\frac{q_s^{'}}{\mu_\lambda h_{f,g}} \right) \left[\frac{\sigma}{g(\rho_\lambda - \rho_v)} \right]^{1/6}$$

1/3

$$T_s - T_{sat} = \frac{0.005(84{,}400 J/kg)9.01^{1.7}}{1100 J/kg.K} \left(\frac{5\times10^4 W/m^2}{440\times10^{-6} kg/m.s \times 84{,}400 J/kg} \right) \times$$

$$\left[\frac{8.1\times10^{-3} kg/s^2}{9.807 m/s^2 (1619.2 - 13.4) kg/m^3} \right]^{1/6} = 15.9°C$$

$$T_s = (15.9 + 57)^\circ C = 72.9°C$$

From the rate equation,

$$T_o = T_s + \frac{q_o^"}{k_s} = 72.9^\circ C + \frac{5\times10^4 W/m^2 \times 0.0025 m}{135 W/m.K} = 73.8°C$$

(b) With the heat rate 90% of the critical heat flux (CHF)

$$q''_{max} = 0.149 hfg \rho v \left[\frac{\sigma g(\rho_l - \rho_v)}{\rho v^2} \right]^{1/4} = 0.149 \times 84{,}400 J/kg \times 13.4 kg/m^3$$

$$\times \left[\frac{8.1 \times 10^{-3} \times 9.807 m/s^2 (1619.2 - 13.4) kg/m^3}{13.4 kg/m^3} \right]^{1/4}$$

$$q''_{max} = 15.5 \times 10^4 W/m^2 \qquad \mathrm{q''_o} = 0.9 q''_{max} = 13.9 \times 10^4 \mathrm{W/m^2}$$

$$\Delta \mathrm{T_e} = \Delta \mathrm{T_e})_a (\mathrm{q''_o}/\mathrm{q''_{o,a}})^{1/3} = 15.9°\mathrm{C} \times 1.41 = 22.4^\circ \mathrm{C} \qquad \mathrm{T_s} = 22.4°\mathrm{C} + 57°\mathrm{C} = 79.4^\circ \mathrm{C}$$

$$\mathrm{T_o} = 79.4\mathrm{C} + \frac{13.9 \times 10^4 \mathrm{W/m^2} \times .0025}{135 \mathrm{W/m.K}} = 82.0^\circ C$$

Comments: Pool boiling is not adequate for many VLSI chip design

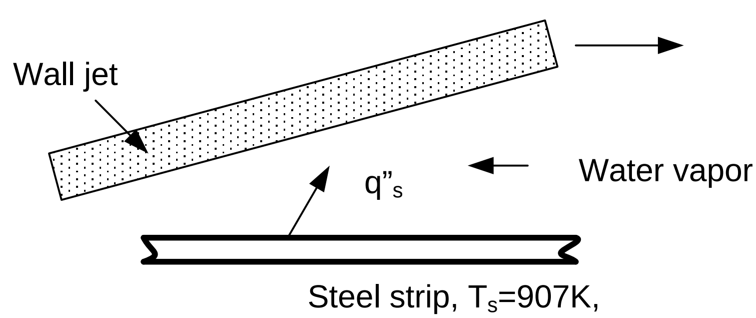
4. Strip steel is made by rolling the strip on a set of rollers in a hot rolling mill. As strip steel leaves the last set of rollers, it is quenched by water jets before being coiled. Because the plate temperatures are high, film boiling is achieved as a result.

   Consider conditions for which the strip steel beneath the vapor blanket is at a temperature of 907K and has an emissivity of 0.35. Neglecting the effects of the strip and jet motions and assuming convection within the film to be approximated by that associated with a large horizontal cylinder of 1-m-diameter, estimate the rate of heat transfer per unit surface area from the strip to the wall.

   Known: Surface temperature and emissivity of strip steel.

   Find: heat flux across vapor blanket.

   Schematic:



   Assumptions: (1) Steady-state conditions, (2) Vapor/jet interface is at T_{sat} for P=1atm, (3) Negligible effect of jet and strip motion.

Properties: Table: saturated Water (100°C 1atm) $\rho_l=1/v_f=957.9 kg/m^3$, $h_{fg}=2257$ KJ/kg: saturated water vapor (T_f=640K): $\rho_v=175.4 kg/m^3$, $c_{p,v}=42$ J/kg.K, $\mu_v=32*10^{-6} N.s/m^2$, $k=0.155 W/m.K$, $\nu_v=0.182*10^{-6} m^2/s$.

Analysis: The heat flux is

$$q_x^{"} = \bar{h}\,\Delta T_e$$

Where $\Delta T_e = 907K - 373K = 534K$

and $$\bar{h}^{-4/3} = \bar{h}_{conv}^{-4/3} + \bar{h}_{rad}\,\bar{h}^{-1/3}$$

with $h_{fg}^{'} = h_{fg} + 0.80 c_{p,v}(T_s - T_{sat}) = 2.02\times10^7$ J/kg

$$\bar{N}u_D = 0.62\left[\frac{9.8 m/s^2 (957.9-175.4) kg/m^3 (2.02\times10^{-7} J/kg)(1m)^3}{0.182\times10^{-6} m^2/s (0.155 W/m.K)(907-373)K}\right]^{1/4} = 6243$$

hence,

$$\bar{h}_{conv} = \bar{N}u_D k_v / D = 6243 W/m^2.K (0.155 W/m.K/1m) = 968 W/m^2.K$$

$$\bar{h}_{rad} = \frac{\varepsilon\sigma(T_s^4 - T_{sat}^4)}{T_s - T_{sat}} = \frac{0.35\times5.67\times10^{-8} W/m^2.k(907^4 - 373^4)K^4}{(907-373)K}$$

$$\bar{h}_{rad} = 24 W/m2.K$$

hence, $$\bar{h} = 968 W/m2.K + (3/4)(24 W/m2.K) = 986 W/m2.K$$

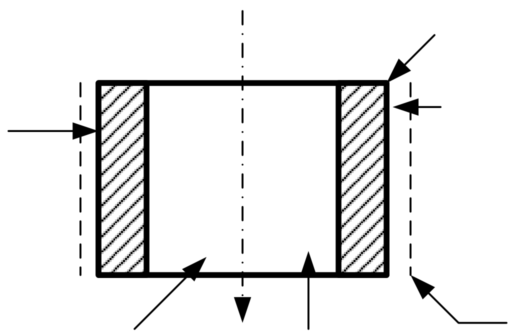
and $$q_s^{"} = 986 W/m^2.k(907-373)K = 5.265\times10^5 W/m^2$$

5. Saturated steam at 0.1 atm condenses with a convection coefficient of 6800 W/m^2.K on the outside of a brass tube having inner and outer diameters of 16.5 and 19mm, respectively. The heat transfer coefficient for water flowing inside the tube is 5200W/m^2.K. Assuming that the mean water temperature is 30°C, calculate the steam condensation rate per unit tube length.

Known: saturated steam condensing on the outside of a brass tube and water flowing on the inside of the tube; convection coefficients are prescribed.

Find: Steam condensation rate per unit length of the tube.

Schematic:



Assumptions: (1) Steady-state conditions.

Properties: Table: Water, vapor (0.1 bar): T_{sat}≈320K, h_{fg}=2390*103J/kg;

Table: Brass ($\bar{T} = (Tm + Tsat)/2 \approx 300K$); k=110W/m.K.

Analysis: The condensation rate per unit length is written as

$$\dot{m}' = q' / h_{fg}' \qquad (1)$$

Where the heat rate follows from equation using overall heat transfer coefficient

$$q' = U_o.\pi D_o(T_{sat} - T_m) \qquad (2)$$

$$U_o = \left[\frac{1}{h_o} + \frac{D_o/2}{k}\lambda n\frac{D_o}{D_i} + \frac{D_o}{D_i}\frac{1}{h_i}\right]^{-1} \qquad (3)$$

$$U_o = \left[\frac{1}{6800W/m^2.K} + \frac{0.0095m}{110W/m.K}\lambda n\frac{19}{16.5} + \frac{19}{16.5}\frac{1}{5200W/m^2.K}\right]^{-1}$$

$$U_o = 147.1\times10^{-6} + 12.18\times10^{-6} + 192.3\times10^{-6}W/m^2.K = 2627W/m^2.K$$

Combining equations, (1) and (2) and substituting numerical values, find

$$\dot{m}' = U_o\pi D_o(T_{sat} - T_m)/h'_{fg}$$

$$\dot{m}' = 2627W/m^2.K\pi(0.019m)(320-303)K/2410\times10^3 J/kg = 1.11\times10^3 kg/s$$

Comments: (1) Note from evaluation of equation. (3) That the thermal resistance of the brass tube is not negligible.

(2) With $Ja = c_{p,\lambda}(T_{sat} - T_s)/h_{f,g}, h'_{fg} = h_{fg}[1 + 0.68Ja]$.note from expression for U_o, that the internal resistance is the largest. Hence, estimate $T_{s,o} \approx T_o - (R_o/\Sigma R)(T_o - T_m) \approx 313K$.Hence

$$h'_{fg} \approx 2390\times10^3 J/kg[1 + 0.68\times4179J/kg.K(320-313)K/2390\times10^3 J/kg]$$

$$h'_{fg} = 2410kJ/kg$$

Where $c_{p,\lambda}$ for water(liquid) is evaluated at $T_f = (T_{s,o} + T_o)/2 \approx 317K$